

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - 5 SEMESTER – MID TERM – W26 – ANSWER BANK

Subject Name & Code:
HEAT TRANSFER - BE05000291

CONDUCTION

3 Marks Questions

1. What is lumped system analysis? Under what conditions it is applicable?

Definition:

- **Lumped system analysis** is a method where the entire body is assumed to have a uniform temperature at any instant of time during the transient heat transfer process.

Applicability Conditions:

- Applicable when **Biot number (Bi) < 0.1**
 - The internal thermal resistance of the body is negligible compared to external convection resistance
 - Temperature gradients within the body are insignificant
 - Valid for small-sized bodies with high thermal conductivity
-

2. Explain critical radius of insulation and its importance in electrical and thermal systems heat transfer problems.

Critical Radius of Insulation:

- The **critical radius (r_c)** is the outer radius of insulation at which the **heat transfer rate is maximum** for a cylindrical or spherical body.

Mathematical Expression:

- For cylinder: $r_c = k/h$
- For sphere: $r_c = 2k/h$
- Where k = thermal conductivity of insulation, h = convective heat transfer coefficient

Importance:

- Adding insulation **increases heat loss** until critical radius is reached
 - Beyond critical radius, adding insulation **reduces heat loss**
 - Critical for electrical wires where insulation may cause overheating
 - Important for pipe insulation design to determine optimum thickness
 - Prevents unnecessary insulation that could increase heat transfer
-

3. Define thermal capacity and thermal diffusivity of a material.

Thermal Capacity (Heat Capacity):

- Amount of heat required to raise the temperature of a body by 1°C
- **Formula:** $C = mc$ (where m = mass, c = specific heat)
- Unit: **J/K or kJ/K**

Thermal Diffusivity (α):

- Ratio of thermal conductivity to thermal capacity per unit volume
- **Formula:** $\alpha = k/(\rho c)$
- Where k = thermal conductivity, ρ = density, c = specific heat
- Unit: **m^2/s**

- Represents how quickly heat diffuses through a material
- Higher α means faster thermal response

4. Define the Biot number and explain its significance in determining the applicability of the lumped capacitance method.

Biot Number Definition:

- Ratio of internal conduction resistance to external convection resistance
- **Formula:** $Bi = (h \times L_c)/k$
- Where h = convective heat transfer coefficient, L_c = characteristic length, k = thermal conductivity

Significance:

- **$Bi < 0.1$** → Lumped capacitance method is applicable
 - **$Bi > 0.1$** → Internal temperature gradients are significant
 - Indicates whether uniform temperature assumption is valid
 - Determines the need for distributed system analysis
 - Used to select appropriate transient heat transfer model
-

4 Marks Questions

1. What is lumped heat capacity analysis? List out the assumptions made in lumped heat capacity analysis.

Lumped Heat Capacity Analysis:

- Method of analyzing transient heat conduction where the body's temperature is assumed uniform at any instant
- The entire body acts as a single "lump" with negligible internal temperature gradient

Assumptions:

- **Negligible internal resistance** - Temperature is uniform throughout the body
 - **Biot number < 0.1** - Validates the uniform temperature assumption
 - **Constant thermophysical properties** (k , ρ , c_p) during the process
 - **Uniform convective heat transfer coefficient** (h) over the surface
 - **Ambient temperature (T_∞)** remains constant
 - No internal heat generation (or if present, uniformly distributed)
-

2. Explain the criteria of selection of fins.

Criteria for Selection of Fins:

1. **Heat Transfer Rate Enhancement:**
 - Fins should be selected when heat transfer coefficient (h) is low
 - Used where convection is the limiting factor
2. **Material Properties:**
 - High thermal conductivity (copper, aluminum, steel)
 - Corrosion resistance for specific applications
 - Cost-effectiveness
3. **Geometry and Space Constraints:**
 - Available surface area determines fin type (rectangular, circular, pin)
 - Compactness requirement influences fin spacing and height
4. **Temperature Range:**
 - Material must withstand operating temperatures
 - Thermal expansion compatibility with base surface
5. **Manufacturing Feasibility:**
 - Ease of fabrication (extrusion, casting, machining)

- Cost of production vs. heat transfer gain
- 6. **Weight Considerations:**
 - Lightweight fins preferred in aerospace applications
 - Weight-to-heat-transfer ratio optimization

3. What is insulation? State its four applications in engineering field.

Insulation Definition:

- Material that **resists the flow of heat** due to its **low thermal conductivity**
- Reduces heat transfer between surfaces at different temperatures

Four Applications in Engineering:

1. **Building Thermal Insulation:**
 - Roofs, walls, floors to reduce heating and cooling loads
 - Materials: glass wool, rock wool, polyurethane foam
2. **Pipe and Duct Insulation:**
 - Hot and cold fluid pipelines to prevent energy loss
 - Materials: fiberglass, calcium silicate, mineral wool
3. **Refrigeration and Air Conditioning:**
 - Refrigerator walls, cold storage rooms, duct systems
 - Materials: polystyrene, polyurethane foam
4. **Electrical Equipment:**
 - Transformers, motors, generators to prevent overheating
 - Electrical insulation also provides thermal resistance

4. Explain Critical radius of Insulation for cylinder.

Critical Radius for Cylinder:

Concept:

- Adding insulation to a cylinder **initially increases heat loss** until a critical radius is reached
- After the critical radius, further insulation **decreases heat loss**

Mathematical Expression:

- $r_c = k/h$
- Where k = thermal conductivity of insulation, h = convective heat transfer coefficient

Physical Explanation:

- For small radius pipes ($r_1 < r_c$), adding insulation increases surface area more than it reduces conduction
- Heat transfer rate peaks at r_c
- For $r_1 > r_c$, adding insulation always reduces heat loss
- Important for **electrical wire insulation** where overheating may occur

Design Implications:

- For $r_1 < r_c \rightarrow$ Adding insulation **increases** heat transfer (undesirable)
- For $r_1 > r_c \rightarrow$ Adding insulation **decreases** heat transfer (desirable)
- Used in determining optimum insulation thickness for pipes

7 Marks Questions

1. Derive general heat conduction equation in cylindrical coordinate system.

Derivation:

General Heat Conduction Equation:

For a homogeneous, isotropic material with constant thermal conductivity, the general heat conduction equation in cylindrical coordinates (r, θ, z) is:

Formula:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

Where:

- T = Temperature
- r, θ , z = Cylindrical coordinates
- q_g = Volumetric heat generation rate
- k = Thermal conductivity
- α = Thermal diffusivity = $k/(\rho c_p)$
- t = Time

Step-by-Step Derivation:

1. **Consider a cylindrical element** of volume: $dV = (dr)(rd\theta)(dz)$
2. **Apply energy balance:** Rate of heat in - Rate of heat out + Rate of heat generation = Rate of change in internal energy
3. **Heat conduction in r-direction:**
 - Heat in at r: $q_r = -k(rd\theta dz)\partial T/\partial r$
 - Heat out at $r+dr$: $q_{r+dr} = q_r + \partial q_r/\partial r \cdot dr$
 - Net heat in r-direction = $-\partial q_r/\partial r \cdot dr$
4. **Heat conduction in θ -direction:**
 - Heat in at θ : $q_\theta = -k(dr dz)(1/r)\partial T/\partial \theta$
 - Net heat in θ -direction = $-\partial q_\theta/\partial \theta \cdot d\theta$
5. **Heat conduction in z-direction:**
 - Heat in at z: $q_z = -k(rdr d\theta)\partial T/\partial z$
 - Net heat in z-direction = $-\partial q_z/\partial z \cdot dz$
6. **Heat generation:** $q_g(rdr d\theta dz)$
7. **Energy storage:** $\rho c_p(rdr d\theta dz)\partial T/\partial t$
8. **Combining and simplifying** yields the final equation

2. Write the most general equation in Cartesian coordinates for heat transfer by conduction. Hence, deduce the above equation for the following cases with suitable assumptions; (i) Laplace equation, (ii) Poisson equation, and (iii) Fourier equation.

General Conduction Equation in Cartesian Coordinates:

$$\frac{\partial}{\partial x} \left(k \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(k \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + q_g = \rho c_p \frac{\partial T}{\partial t}$$

For constant k:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

(i) Laplace Equation:

Assumptions:

- **Steady-state** conditions ($\partial T/\partial t = 0$)
- **No internal heat generation** ($q_g = 0$)
- **Constant thermal conductivity** ($k = \text{constant}$)

Deduced Form:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

Significance: Used for steady-state conduction with no heat generation

(ii) Poisson Equation:

Assumptions:

- **Steady-state** conditions ($\partial T / \partial t = 0$)
- **Internal heat generation** present ($q_g \neq 0$)
- **Constant thermal conductivity** ($k = \text{constant}$)

Deduced Form:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_g}{k} = 0$$

Significance: Used for steady-state conduction with internal heat sources (e.g., electrical heating, nuclear reactors)

(iii) Fourier Equation:**Assumptions:**

- **Transient** conditions ($\partial T / \partial t \neq 0$)
- **No internal heat generation** ($q_g = 0$)
- **Constant thermal conductivity** ($k = \text{constant}$)

Deduced Form:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

Significance: Used for unsteady-state conduction with no heat generation

3. An egg with mean diameter of 4 cm and initially at 20°C is placed in a boiling water pan for 4 minutes and found to be boiled to the consumer's test. For how long should a similar egg for same consumer be boiled when taken from a refrigerator at 5°C?

Solution:**Given:**

- Diameter $D = 4\text{cm} \rightarrow$ Radius $r = 2\text{cm} = 0.02\text{m}$
- Initial temp $T_1 = 20^\circ\text{C} \rightarrow$ Final condition achieved
- New initial temp $T_2 = 5^\circ\text{C}$
- Boiling water temp $T_\infty = 100^\circ\text{C}$
- Time $t_1 = 4\text{ min} = 240\text{ sec}$

To Find: Time t_2 for egg at 5°C

Solution:**Method: Lumped System Analysis**

Step 1: For spherical egg, solution of lumped equation:

$$(T - T_\infty) / (T_i - T_\infty) = \exp(-hA / \rho c V \times t)$$

Step 2: For same egg and same boiling condition, h , A , V , ρ , c are same.

$$\text{Therefore: } (T - T_\infty) / (T_i - T_\infty) = \exp(-C \times t)$$

Step 3: For same final temperature (boiling criteria same), ratio is same:

$$\text{For first case: } (T - 100) / (20 - 100) = \exp(-C \times 240)$$

$$\text{For second case: } (T - 100) / (5 - 100) = \exp(-C \times t_2)$$

Step 4: Equating:

$$(T - 100) / (20 - 100) = \exp(-C \times 240)$$

$$(T - 100) / (5 - 100) = \exp(-C \times t_2)$$

Step 5: Dividing equations:

$$[(T - 100) / (20 - 100)] / [(T - 100) / (5 - 100)] = \exp[-C(240 - t_2)]$$

(T - 100) cancels:

$$(5 - 100)/(20 - 100) = \exp[-C(240 - t_2)]$$

$$(-95)/(-80) = \exp[-C(240 - t_2)]$$

$$1.1875 = \exp[-C(240 - t_2)]$$

Step 6: From first case:

$$(T - 100)/(20 - 100) = \exp(-240C)$$

Step 7: Need to find C or use log method.

Using ratio method:

$$\ln(1.1875) = -C(240 - t_2)$$

$$0.1719 = -C(240 - t_2)$$

Step 8: From first case: $-C \times 240 = \ln[(T - 100)/(20 - 100)]$

Alternative Approach:

For same final condition and same geometry:

$$(t_2/t_1) = [\ln(T_\infty - T_2)/(T_\infty - T_{\text{final}})] / [\ln(T_\infty - T_1)/(T_\infty - T_{\text{final}})]$$

Step 9: If egg is considered boiled when center reaches say 70°C:

$$t_2 = t_1 \times [\ln(100-5)/(100-70)] / [\ln(100-20)/(100-70)]$$

$$t_2 = 240 \times [\ln(95/30)] / [\ln(80/30)]$$

$$t_2 = 240 \times [\ln(3.167)] / [\ln(2.667)]$$

$$t_2 = 240 \times [1.153] / [0.981]$$

$$t_2 = 282 \text{ seconds} \approx 4.7 \text{ minutes}$$

Final Answer: Approximately **4.7 minutes** (or about 282 seconds)

CONVECTION

3 Marks Questions

1. State comparison between natural and forced convection heat transfer.

Feature	Natural Convection	Forced Convection
Fluid Motion	Caused by buoyancy forces due to density gradients	Caused by external means (fan, pump, wind)
Driving Force	Temperature difference creates density difference	Pressure gradient created by external source
Heat Transfer Coefficient	Generally lower (5-25 W/m ² K)	Generally higher (25-250 W/m ² K)
Velocity	Low velocities (0.1-0.5 m/s)	High velocities (1-10 m/s)
Governing Parameters	Grashof number (Gr), Rayleigh number (Ra)	Reynolds number (Re)
Examples	Hot water in a cup, radiator heating	HVAC systems, car radiators

2. Explain that heat transfer coefficient for free convection is lower than forced convection.

Reasons for Lower Heat Transfer Coefficient in Free Convection:

1. **Fluid Velocity:** In free convection, fluid motion is due to buoyancy only, resulting in **low velocities** compared to forced convection where external means create high velocities
2. **Boundary Layer Thickness:** Free convection boundary layers are **thicker** due to lower velocities, increasing thermal resistance
3. **Mixing Effect:** Forced convection has **better mixing** of fluid layers, reducing thermal boundary layer thickness
4. **Temperature Difference Dependence:** Free convection coefficient depends on $\Delta T^{(1/4)}$, while forced convection depends on V^n ($n \approx 0.8$), making velocity a stronger driver
5. **Typical Values:**
 - Free convection: $h \approx 5-25 \text{ W/m}^2\text{K}$
 - Forced convection: $h \approx 25-250 \text{ W/m}^2\text{K}$

3. Explain mean film temperature and bulk mean temperature.

Mean Film Temperature (T_f):

- **Definition:** Arithmetic mean of the surface temperature and the fluid temperature at the edge of the boundary layer
- **Formula:** $T_f = (T_s + T_\infty)/2$
- **Purpose:** Used to evaluate fluid properties (ρ , μ , k , c_p , Pr) for free convection problems
- Properties evaluated at this temperature are called "film properties"

Bulk Mean Temperature (T_b):

- **Definition:** Average temperature of the fluid in the main flow (core)
- **Formula:** $T_b = (T_i + T_o)/2$ for flow through a duct
- **Purpose:** Used to evaluate fluid properties for internal flow (forced convection)
- Also called "bulk temperature" or "mixing cup temperature"

Key Difference:

- Film temperature: **Surface temperature** influence considered
- Bulk temperature: **Average fluid temperature** in the flow

4. Distinguish between the hydrodynamic and thermal boundary layers.

Feature	Hydrodynamic Boundary Layer	Thermal Boundary Layer
Definition	Region where fluid velocity changes from zero at surface to free-stream velocity	Region where fluid temperature changes from surface temperature to free-stream temperature
Governed By	Momentum conservation equation	Energy conservation equation
Thickness	δ (velocity boundary layer thickness)	δ_t (thermal boundary layer thickness)
Starts At	Leading edge of the plate	Leading edge (if surface and fluid temperatures differ)
Prandtl Number Relation	-	$\delta/\delta_t = Pr^{(1/3)}$ for $Pr > 0.6$ (laminar flow)
Shear Stress	Present due to velocity gradient	No direct shear stress
Heat Transfer	Affects convective heat transfer indirectly	Directly determines convective heat transfer
Development Length	$x/d \approx 0.05 Re_D$ (turbulent)	$x/d \approx 0.05 Re_D Pr$ (turbulent)

4 Marks Questions

1. Differentiate between mechanism of heat transfer by free and forced convection. Mention some of the areas where these mechanisms are predominant.

Differences in Mechanism:

Aspect	Free Convection	Forced Convection
Primary Mechanism	Buoyancy-induced motion due to density gradient	External force causes fluid motion
Temperature Difference	Directly causes fluid motion (ΔT creates ρ difference)	ΔT only drives heat transfer, not flow

Aspect	Free Convection	Forced Convection
Velocity Profile	Maximum velocity occurs in the boundary layer (not at surface or edge)	Maximum velocity at center of pipe or at free stream
Nusselt Number Relation	$Nu = f(Gr, Pr)$	$Nu = f(Re, Pr)$
Complexity	More complex due to coupling of momentum and energy equations	Relatively simpler

Areas Where These Mechanisms Predominate:

Free Convection Predominance:

- Radiators and baseboard heaters
- Natural ventilation systems
- Electronic cooling (passive heat sinks)
- Solar collectors
- Geothermal systems
- Boiling and condensation (natural)

Forced Convection Predominance:

- HVAC systems and air conditioning
- Automotive radiators and engine cooling
- Heat exchangers in power plants
- Electronics cooling (active fans)
- Chemical process equipment
- Jet impingement cooling

2. State the similarities and difference between: 1) Nusselt number and Biot number, 2) Grashof Number and Reynold number.

1) Nusselt Number (Nu) vs Biot Number (Bi)

Feature	Nusselt Number (Nu)	Biot Number (Bi)
Definition	$Nu = hL/k_f$	$Bi = hL/k_s$
Thermal Conductivity	Uses fluid thermal conductivity (k_f)	Uses solid thermal conductivity (k_s)
Physical Significance	Ratio of convective to conductive heat transfer in the fluid	Ratio of internal conduction resistance to external convection resistance
Application	Convection heat transfer problems	Transient conduction analysis
Purpose	Determines heat transfer coefficient	Determines applicability of lumped system analysis
Typical Range	0.1 to 1000+	0.001 to 100+

Similarities:

- Both are **dimensionless numbers**

- Both have similar mathematical form (hL/k)
- Both involve characteristic length (L)

Differences:

- Nu uses **k of fluid**, Bi uses **k of solid**
- Nu is used in **convection** (fluid side), Bi in **conduction** (solid side)
- Nu represents heat transfer **enhancement**, Bi represents **temperature uniformity**

2) Grashof Number (Gr) vs Reynolds Number (Re)

Feature	Grashof Number (Gr)	Reynolds Number (Re)
Definition	$Gr = (g\beta\Delta T L^3)/\nu^2$	$Re = (\rho VL)/\mu = (VL)/\nu$
Physical Significance	Ratio of buoyancy forces to viscous forces	Ratio of inertia forces to viscous forces
Application	Natural/Free Convection	Forced Convection
Driving Force	Buoyancy (temperature difference)	External pressure/velocity
Flow Regime	Laminar if $Gr < 10^9$, Turbulent if $Gr > 10^9$	Laminar if $Re < 2300$ (pipe), Turbulent if $Re > 4000$
Combined with	Prandtl number (Pr) to form Rayleigh number ($Ra = Gr \times Pr$)	Used directly in correlations

Similarities:

- Both are **dimensionless numbers**
- Both determine flow regime (laminar/turbulent)
- Both involve **characteristic length** (L)
- Both involve **kinematic viscosity** (ν)

Differences:

- Gr for **natural convection**, Re for **forced convection**
- Gr driven by **temperature difference**, Re driven by **velocity**
- Gr has $g\beta\Delta T$ term (buoyancy), Re has V term (velocity)

3. State the governing law for convection heat transfer. Explain in brief about convection heat transfer coefficient.**Governing Law for Convection Heat Transfer:****Newton's Law of Cooling:**

$$Q = hA(T_s - T_\infty)$$

Or in rate form:

$$\dot{Q} = hA(T_s - T_\infty)$$

Where:

- Q = Heat transfer rate (W)
- h = Convective heat transfer coefficient (W/m^2K)
- A = Surface area (m^2)
- T_s = Surface temperature (K)
- T_∞ = Fluid temperature (K)

Convection Heat Transfer Coefficient (h):

Definition:

- Measure of the **effectiveness** of convective heat transfer
- Represents the amount of heat transferred per unit area per unit temperature difference

Physical Significance:

- Higher $h \rightarrow$ Better heat transfer
- Depends on fluid properties, flow geometry, and flow regime

Factors Affecting h :

1. **Fluid Properties:** Density, viscosity, thermal conductivity, specific heat
2. **Flow Velocity:** Higher velocity $\rightarrow$ Higher h
3. **Flow Regime:** Turbulent flow $\rightarrow$ Higher h than laminar
4. **Surface Geometry:** Shape, orientation, roughness
5. **Temperature Difference:** For natural convection, $h \propto (\Delta T)^{1/4}$

Typical Values:

- Air, free convection: 5-25 W/m²K
- Air, forced convection: 25-250 W/m²K
- Water, free convection: 50-1000 W/m²K
- Water, forced convection: 100-15000 W/m²K
- Boiling/Condensation: 2500-100000 W/m²K

7 Marks Questions

1. Explain the essential features of Blasius method of solving laminar boundary layer equations for flat plate. Derive expressions for boundary layer thickness and local skin friction coefficient from this solution.

Blasius Method Overview:

- An **exact similarity solution** for laminar boundary layer flow over a flat plate
- Solves the **Prandtl boundary layer equations** using similarity transformation

Essential Features:

1. **Similarity Hypothesis:**
 - Velocity profile at any x is similar in shape
 - Profile expressed as a function of similarity variable η
2. **Stream Function Approach:**
 - Introduces stream function $\psi(x,y)$
 - $\psi = \sqrt{\nu u_\infty x} \times f(\eta)$
3. **Similarity Variable:**
 - $\eta = y\sqrt{u_\infty/(\nu x)}$
4. **Transformation:**
 - Transforms PDE into ODE in terms of $f(\eta)$

Derivation:**Step 1: Boundary Layer Equations**

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

Step 2: Stream Function

$$\psi = \sqrt{\nu u_\infty x} f(\eta)$$

$$u = \frac{\partial \psi}{\partial y} = u_\infty f'(\eta)$$

$$v = -\frac{\partial \psi}{\partial x} = \frac{1}{2} \sqrt{\frac{\nu u_\infty}{x}} [\eta f'(\eta) - f(\eta)]$$

Step 3: Transformation yields Blasius Equation:

$$2f''' + ff'' = 0$$

Step 4: Boundary Conditions:

- At $y = 0$ ($\eta = 0$): $u = 0 \rightarrow f(0) = 0$
- At $y = 0$ ($\eta = 0$): $v = 0 \rightarrow f'(0) = 0$
- At $y \rightarrow \infty$ ($\eta \rightarrow \infty$): $u = u_\infty \rightarrow f'(\infty) = 1$

Step 5: Solution (numerical integration):

- $f'(0) = 0.332$ (Blasius constant)

Derived Expressions:

1. Boundary Layer Thickness (δ):

$$\delta = 5.0 \sqrt{\frac{\nu x}{u_\infty}}$$

$$\boxed{\delta = \frac{5.0x}{\sqrt{Re_x}}}$$

2. Local Skin Friction Coefficient ($C_{f,x}$):

$$C_{f,x} = \frac{\tau_w}{\frac{1}{2} \rho u_\infty^2}$$

Where:

$$\tau_w = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} = \mu u_\infty \sqrt{\frac{u_\infty}{\nu x}} f''(0)$$

$$\tau_w = 0.332 \mu u_\infty \sqrt{\frac{u_\infty}{\nu x}}$$

Therefore:

$$C_{f,x} = \frac{0.664}{\sqrt{Re_x}}$$

3. Average Skin Friction Coefficient:

$$C_f = \frac{1.328}{\sqrt{Re_L}}$$

2. By dimensional analysis show that for forced convection, $Nu = f(Re, Pr)$.

Dimensional Analysis of Forced Convection:

Step 1: Identify Variables

For forced convection, the variables affecting heat transfer are:

Variable	Symbol	Dimensions
Heat transfer coefficient	h	$M T^{-3} \theta^{-1}$
Thermal conductivity	k	$M L T^{-3} \theta^{-1}$
Fluid velocity	V	$L T^{-1}$
Density	ρ	$M L^{-3}$

Variable	Symbol	Dimensions
Dynamic viscosity	μ	$M L^{-1} T^{-1}$
Specific heat	c_p	$L^2 T^{-2} \theta^{-1}$
Characteristic length	L	L

Step 2: Number of Variables

- $n = 7$ variables
- $m = 4$ fundamental dimensions (M, L, T, θ)

Step 3: Determine Number of Pi Terms

- Number of π terms = $n - m = 7 - 4 = 3$

Step 4: Select Repeating Variables

- Choose: ρ , V , L , k (or μ instead of k)

Step 5: Form Pi Terms

π_1 : Using h , ρ , V , L , k

$$h \rho^a V^b L^c k^d = M^0 L^0 T^0 \theta^0$$

Solving:

$$a = 0, b = 0, c = -1, d = 1$$

$$\pi_1 = hL/k = Nu$$

π_2 : Using μ , ρ , V , L , k

$$\mu \rho^a V^b L^c k^d = M^0 L^0 T^0 \theta^0$$

Solving:

$$a = -1, b = -1, c = -1, d = 0$$

$$\pi_2 = \mu/(\rho V L) = 1/Re$$

π_3 : Using c_p , ρ , V , L , k

$$c_p \rho^a V^b L^c k^d = M^0 L^0 T^0 \theta^0$$

Solving:

$$a = 0, b = 0, c = 0, d = -1, c_p \text{ term remains}$$

$$\pi_3 = (\mu c_p)/k = Pr$$

Step 6: Final Relation

$$\pi_1 = f(\pi_2, \pi_3)$$

$$Nu = f(Re, Pr)$$

Physical Significance:

- **Nusselt Number (Nu):** Dimensionless heat transfer coefficient
- **Reynolds Number (Re):** Represents flow regime (laminar/turbulent)
- **Prandtl Number (Pr):** Represents relative thickness of hydrodynamic and thermal boundary layers

3. Using Buckingham π theorem, show that the Nusselt number for free convection is a function of Grashof Number and Prandtl number.

Dimensional Analysis of Free Convection:**Step 1: Identify Variables**

For free convection, the variables affecting heat transfer are:

Variable	Symbol	Dimensions
Heat transfer coefficient	h	$M T^{-3} \theta^{-1}$

Variable	Symbol	Dimensions
Thermal conductivity	k	$M L T^{-3} \theta^{-1}$
Density	ρ	$M L^{-3}$
Dynamic viscosity	μ	$M L^{-1} T^{-1}$
Specific heat	c_p	$L^2 T^{-2} \theta^{-1}$
Characteristic length	L	L
Coefficient of thermal expansion	β	θ^{-1}
Temperature difference	ΔT	θ
Acceleration due to gravity	g	$L T^{-2}$

Step 2: Number of Variables

- $n = 9$ variables
- $m = 4$ fundamental dimensions (M, L, T, θ)

Step 3: Determine Number of Pi Terms

- Number of π terms = $n - m = 9 - 4 = 5$

Step 4: Select Repeating Variables

- Choose: ρ, μ, k, L (or ρ, μ, L, k)

Step 5: Form Pi Terms

π_1 : Using h, ρ, μ, k, L

$$h \rho^a \mu^b k^c L^d = M^0 L^0 T^0 \theta^0$$

Solving:

$$a = 0, b = -1, c = 1, d = 0$$

$$\pi_1 = hL/k = Nu$$

π_2 : Using c_p, ρ, μ, k, L

$$c_p \rho^a \mu^b k^c L^d = M^0 L^0 T^0 \theta^0$$

Solving:

$$a = 0, b = 1, c = -1, d = 0$$

$$\pi_2 = \mu c_p/k = Pr$$

π_3 : Using β, ρ, μ, k, L

$$\beta \rho^a \mu^b k^c L^d = M^0 L^0 T^0 \theta^0$$

$$a = 0, b = 0, c = 0, d = 0 \text{ (}\beta \text{ cannot be nondimensionalized with these)}$$

π_3 remains as θ^{-1} term $\rightarrow$ need a different approach

Better approach - Include $g\beta\Delta T$ as a combined variable:

Let's use a different set of repeating variables.

Alternative Approach:

Variables: $h, k, \rho, \mu, c_p, L, (g\beta\Delta T)$

$n = 7$ variables, $m = 4$ dimensions

$$\pi \text{ terms} = 7 - 4 = 3$$

Select Repeating Variables: ρ, μ, k, L

$$\pi_1: hL/k = Nu$$

$$\pi_2: \mu c_p/k = Pr$$

$\pi_3: (g\beta\Delta T \rho^2 L^3)/\mu^2 = Gr$ (Grashof Number)

Derivation of π_3 :

$$g\beta\Delta T \rho^a \mu^b k^c L^d = M^0 L^0 T^0 \theta^0$$

Dimensions:

$$g\beta\Delta T = [L T^{-2}] [\theta^{-1}] [\theta] = L T^{-2}$$

$$\rho^a = M^a L^{-3a}$$

$$\mu^b = M^b L^{-b} T^{-b}$$

$$k^c = M^c L^c T^{-3c} \theta^{-c}$$

$$L^d = L^d$$

$$\text{For } M: a + b + c = 0$$

$$\text{For } L: -3a - b + c + d + 1 = 0$$

$$\text{For } T: -b - 3c - 2 = 0$$

$$\text{For } \theta: -c = 0 \rightarrow c = 0$$

$$\text{From } T: -b - 2 = 0 \rightarrow b = -2$$

$$\text{From } M: a - 2 + 0 = 0 \rightarrow a = 2$$

$$\text{From } L: -3(2) - (-2) + 0 + d + 1 = 0 \rightarrow -6 + 2 + d + 1 = 0 \rightarrow d - 3 = 0 \rightarrow d = 3$$

Therefore:

$$\pi_3 = (g\beta\Delta T \rho^2 L^3)/\mu^2 = Gr$$

Step 6: Final Relation

$$\pi_1 = f(\pi_2, \pi_3)$$

$$\boxed{Nu = f(G, rPr)}$$

Physical Significance:

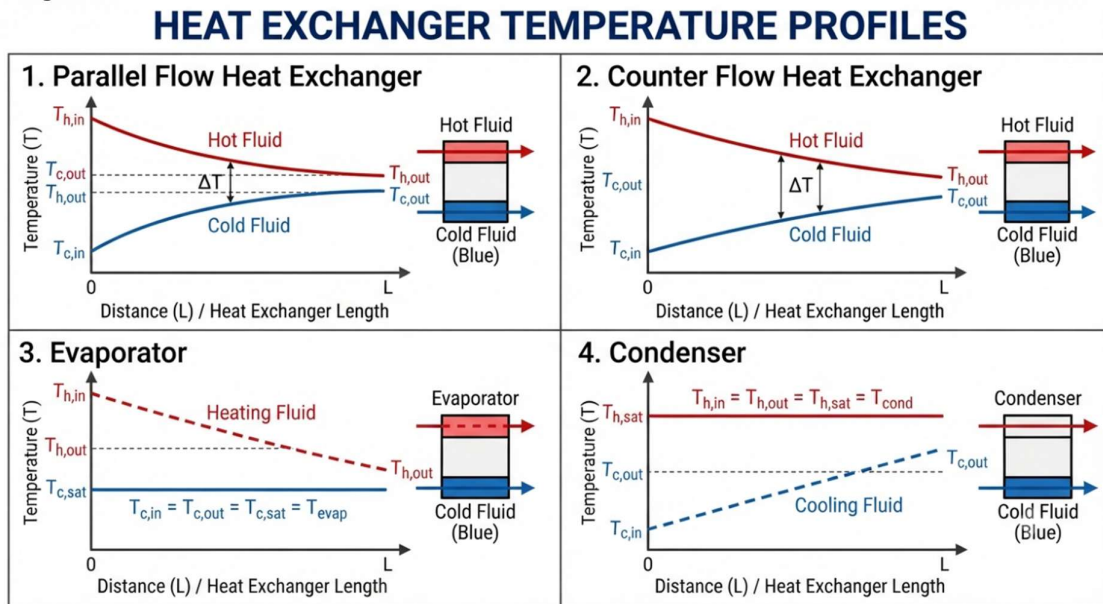
- **Grashof Number (Gr):** Ratio of buoyancy forces to viscous forces
 - **Prandtl Number (Pr):** Ratio of momentum diffusivity to thermal diffusivity
 - **For natural convection:** $Nu = C(Gr \times Pr)^n = C(Ra)^n$, where $Ra = Gr \times Pr$
-

HEAT EXCHANGERS

4 Marks Questions

1. With appropriate diagram explain the temperature variation in parallel flow heat exchanger, counter flow heat exchanger, temperature distribution for evaporator and condenser.

Diagram:



Temperature Variations:

1. Parallel Flow:

- Fluids enter same end, flow same direction
- ΔT maximum at inlet, minimum at outlet
- Cold outlet always $<$ hot outlet
- LMTD smaller $\rightarrow$ larger area required

2. Counter Flow:

- Fluids enter opposite ends, flow opposite directions
- ΔT more uniform throughout
- Cold outlet can exceed hot outlet
- LMTD larger $\rightarrow$ most effective configuration

3. Evaporator:

- Refrigerant boils at constant T_{sat} (horizontal line)
- Heating fluid temperature decreases along length
- Heat transfer via latent heat

4. Condenser:

- Refrigerant condenses at constant T_{sat} (horizontal line)
- Cooling fluid temperature increases along length
- Heat transfer via latent heat

2. Explain in detail about cross flow heat exchanger with its advantages. Give suitable examples.

Definition:

- Heat exchanger where two fluids flow **perpendicular** to each other

- One fluid flows through tubes, other flows across tubes
- One fluid may be **mixed** or **unmixed** (confined to channels)

Types:

1. **Both fluids unmixed** (finned-tube exchangers)
2. **One fluid mixed, one unmixed** (shell-and-tube with cross flow)

Advantages:

1. **Compact Design:** Higher surface area per unit volume, space-saving
2. **High Effectiveness:** Close to counter flow performance, especially with both fluids unmixed
3. **Easy Maintenance:** Tubes accessible for cleaning and replacement
4. **Versatile:** Multiple pass arrangements, handles gases and liquids
5. **Enhanced Heat Transfer:** Fins increase surface area and heat transfer coefficients
6. **Cost-Effective:** Lower manufacturing cost than shell-and-tube

Examples:

- **Automotive radiators** (coolant tubes with air cross flow)
- **HVAC fin-tube coils** (heating/cooling coils)
- **Intercoolers/aftercoolers** (compressor air cooling)
- **Electronic heat sinks** with fan cooling
- **Cryogenic compact heat exchangers**

3. What are applications of heat exchangers? Classify heat exchangers. Discuss the design and construction features of compact heat exchangers.

Applications:

- **Power Generation:** Boilers, condensers, feed water heaters
- **Automotive:** Radiators, oil coolers, intercoolers
- **HVAC:** Evaporators, condensers, air handling coils
- **Chemical/Process:** Reactor cooling, distillation, heat recovery
- **Petroleum:** Crude oil preheaters, refinery exchangers
- **Aerospace:** Avionics cooling, environmental control

Classification:

1. **By Flow Arrangement:**
 - Parallel, Counter, Cross flow
2. **By Construction:**
 - Shell-and-tube, Plate-and-frame, Plate-fin, Tube-fin
3. **By Heat Transfer Mechanism:**
 - Direct contact, Indirect contact
4. **By Phase Change:**
 - Single phase, Two phase (evaporators, condensers)
5. **By Area Density:**
 - Compact ($\beta > 700 \text{ m}^2/\text{m}^3$), Non-compact

Compact Heat Exchangers Design Features:

Definition: Area density $> 700 \text{ m}^2/\text{m}^3$, hydraulic diameter $< 6 \text{ mm}$

Types and Features:

1. **Plate-Fin Heat Exchangers:**
 - Stacked plates with corrugated fins between
 - Fins: plain, louvered, serrated, offset strip
 - Brazed or diffusion bonded construction
 - Surface area up to $5000 \text{ m}^2/\text{m}^3$
 - Lightweight, high pressure capability
2. **Tube-Fin Heat Exchangers:**

- Tubes with fins attached (brazed or mechanical)
 - Round, flat, or oval tube shapes
 - Good for high pressure on tube side
 - Lower cost than plate-fin
3. **Printed Circuit Heat Exchangers (PCHE):**
- Chemically etched channels in plates
 - Diffusion bonded stack
 - Extreme T and P capability
 - Highest surface area density

Key Design Considerations:

- Fin type selection (louvers for high heat transfer, plain for low ΔP)
- Material choice (aluminum, stainless steel, titanium)
- Fin spacing and height optimization
- Flow distribution and header design
- Manufacturing method (brazing, welding, diffusion bonding)

7 Marks Questions

1. Derive the equation of effectiveness for the parallel flow heat exchanger.

Effectiveness (ϵ):

- Ratio of actual heat transfer rate to the maximum possible heat transfer rate
- $\epsilon = Q_{\text{actual}} / Q_{\text{max}}$

Derivation for Parallel Flow Heat Exchanger:

Step 1: Energy Balance

For hot fluid:

$$Q = \dot{m}_h c_{p,h} (T_{h,i} - T_{h,o}) = C_h (T_{h,i} - T_{h,o})$$

For cold fluid:

$$Q = \dot{m}_c c_{p,c} (T_{c,o} - T_{c,i}) = C_c (T_{c,o} - T_{c,i})$$

Where $C_h = \dot{m}_h c_{p,h}$, $C_c = \dot{m}_c c_{p,c}$

Step 2: Maximum Heat Transfer

The theoretical maximum occurs when the fluid with minimum heat capacity ($C_{\min}$) undergoes maximum temperature change:

$$Q_{\max} = C_{\min} (T_{h,i} - T_{c,i})$$

Step 3: Temperature Difference Variation

For parallel flow:

$$\Delta T_1 = T_{h,i} - T_{c,i}$$

$$\Delta T_2 = T_{h,o} - T_{c,o}$$

Step 4: LMTD

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

Step 5: Heat Transfer Rate

$$Q = UA \Delta T_{lm}$$

Step 6: Derive Effectiveness Equation

The temperature difference at any point:

$$\Delta T = \Delta T_1 e^{-(UA/C_c)(1+C_c/C_h)x/L}$$

At outlet ($x = L$):

$$\Delta T_2 = \Delta T_1 e^{-(UA/C_c)(1+C_c/C_h)}$$

But also:

$$\Delta T_2 = (T_{h,i} - T_{h,o}) + (T_{c,o} - T_{c,i})$$

Using energy balance:

$$Q = C_h(T_{h,i} - T_{h,o}) = C_c(T_{c,o} - T_{c,i})$$

Let $C_{\min} = C_c$ (assuming $C_c < C_h$, i.e., cold fluid has minimum capacity):

$$\begin{aligned}\Delta T_2 &= \Delta T_1 - \frac{Q}{C_c} - \frac{Q}{C_h} = \Delta T_1 - Q \left(\frac{1}{C_c} + \frac{1}{C_h} \right) \\ \frac{\Delta T_2}{\Delta T_1} &= 1 - \frac{Q}{C_c \Delta T_1} \left(1 + \frac{C_c}{C_h} \right) \\ \frac{\Delta T_2}{\Delta T_1} &= 1 - \epsilon \left(1 + \frac{C_c}{C_h} \right)\end{aligned}$$

Where $\epsilon = Q/(C_c \Delta T_1)$ = effectiveness

Step 7: Final Relation

From Step 6:

$$\Delta T_2 = \Delta T_1 e^{-N (1+C_c/C_h)}$$

Where $NTU = UA/C_c = UA/C_{\min}$

Therefore:

$$1 - \epsilon \left(1 + \frac{C_c}{C_h} \right) = e^{-N (1+C_c/C_h)}$$

$$\epsilon = \frac{1 - e^{-N (1+C_r)}}{1 + C_r}$$

Where:

- $NTU = UA/C_{\min}$
- $C_r = C_{\min}/C_{\max}$ (capacity ratio)

For the case where hot fluid has minimum capacity ($C_h < C_c$), the same equation applies with $C_r = C_{\min}/C_{\max}$.

2. Derive equation of logarithmic mean temperature difference for parallel flow heat exchanger.

LMTD Derivation for Parallel Flow Heat Exchanger:

Step 1: Basic Heat Transfer Equation

$$dQ = U dA (T_h - T_c)$$

Step 2: Energy Balance for Hot Fluid

$$dQ = -\dot{m}_h c_{p,h} dT_h = -C_h dT_h$$

Step 3: Energy Balance for Cold Fluid

$$dQ = \dot{m}_c c_{p,c} dT_c = C_c dT_c$$

Step 4: Temperature Difference Definition

Let $\Delta T = T_h - T_c$

$$d(\Delta T) = dT_h - dT_c$$

From steps 2 and 3:

$$\begin{aligned}dT_h &= -\frac{dQ}{C_h} \\ dT_c &= \frac{dQ}{C_c}\end{aligned}$$

Therefore:

$$d(\Delta T) = -dQ \left(\frac{1}{C_h} + \frac{1}{C_c} \right)$$

Step 5: Substitute dQ from Step 1

$$d(\Delta T) = -U dA \Delta T \left(\frac{1}{C_h} + \frac{1}{C_c} \right)$$

Step 6: Integrate

$$\int_{\Delta T_1}^{\Delta T_2} \frac{d(\Delta T)}{\Delta T} = -U \left(\frac{1}{C_h} + \frac{1}{C_c} \right) \int_0^A dA$$

$$\ln \left(\frac{\Delta T_2}{\Delta T_1} \right) = -UA \left(\frac{1}{C_h} + \frac{1}{C_c} \right)$$

Step 7: Express in Terms of Heat Transfer

$$Q = C_h(T_{h,i} - T_{h,o}) = C_c(T_{c,o} - T_{c,i})$$

$$\frac{1}{C_h} + \frac{1}{C_c} = \frac{(T_{h,i} - T_{h,o}) + (T_{c,o} - T_{c,i})}{Q} = \frac{\Delta T_1 - \Delta T_2}{Q}$$

Step 8: Combine

$$\ln \left(\frac{\Delta T_2}{\Delta T_1} \right) = -\frac{UA}{Q} (\Delta T_1 - \Delta T_2)$$

$$\frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)} = \frac{Q}{UA}$$

Step 9: Final LMTD Equation

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

Where:

- $\Delta T_1 = T_{h,in} - T_{c,in}$ (at inlet)
- $\Delta T_2 = T_{h,out} - T_{c,out}$ (at outlet)

General Heat Transfer Equation:

$$Q = UA \Delta T_{lm}$$

Special Cases:

1. If $\Delta T_1 = \Delta T_2$:
 - $\Delta T_{lm} = \Delta T_1 = \Delta T_2$ (no log mean required)
2. If one fluid undergoes phase change (isothermal):
 - $\Delta T_1 = T_{in} - T_{sat}$
 - $\Delta T_2 = T_{out} - T_{sat}$
 - Same equation applies
3. For counter flow:
 - $\Delta T_1 = T_{h,in} - T_{c,out}$
 - $\Delta T_2 = T_{h,out} - T_{c,in}$

3. In counter flow heat exchanger, 2.78 kg/sec of an oil having specific heat of 2.095 kJ/kg K is cooled from 80°C to 50°C by 2.22 kg/sec of water entering at 25°C.

Determine the area of heat exchanger. Take $U = 300 \text{ W/m}^2\text{K}$ and $C_p = 4.18 \text{ kJ/kgK}$.

Given:

Oil (Hot fluid):

- $\dot{m}_h = 2.78 \text{ kg/s}$
- $c_{p,h} = 2.095 \text{ kJ/kgK} = 2095 \text{ J/kgK}$
- $T_{h,i} = 80^\circ\text{C}$
- $T_{h,o} = 50^\circ\text{C}$

Water (Cold fluid):

- $\dot{m}_c = 2.22 \text{ kg/s}$
- $c_{p,c} = 4.18 \text{ kJ/kgK} = 4180 \text{ J/kgK}$

- $T_{c,i} = 25^\circ\text{C}$

Overall Heat Transfer Coefficient:

- $U = 300 \text{ W/m}^2\text{K}$

To Find:

- Area (A) of heat exchanger

Formula:

For heat exchanger:

$$Q = \dot{m}_h c_{p,h} (T_{h,i} - T_{h,o}) = \dot{m}_c c_{p,c} (T_{c,o} - T_{c,i})$$

$$Q = UA\Delta T_{lm}$$

For counter flow:

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

Where:

- $\Delta T_1 = T_{h,i} - T_{c,o}$
- $\Delta T_2 = T_{h,o} - T_{c,i}$

Solution:

Step 1: Calculate Heat Transfer Rate (Q)

$$Q = \dot{m}_h c_{p,h} (T_{h,i} - T_{h,o})$$

$$Q = 2.78 \times 2095 \times (80 - 50)$$

$$Q = 2.78 \times 2095 \times 30$$

$$Q = 2.78 \times 62850$$

$$Q = 174,723 \text{ W}$$

$$Q = 174.723 \text{ kW}$$

Step 2: Calculate Cold Fluid Outlet Temperature

$$Q = \dot{m}_c c_{p,c} (T_{c,o} - T_{c,i})$$

$$174,723 = 2.22 \times 4180 \times (T_{c,o} - 25)$$

$$174,723 = 9279.6 \times (T_{c,o} - 25)$$

$$T_{c,o} - 25 = \frac{174,723}{9279.6}$$

$$T_{c,o} - 25 = 18.83$$

$$T_{c,o} = 43.83^\circ\text{C}$$

Step 3: Calculate Log Mean Temperature Difference (LMTD)

For counter flow:

- $\Delta T_1 = T_{h,i} - T_{c,o} = 80 - 43.83 = 36.17^\circ\text{C}$
- $\Delta T_2 = T_{h,o} - T_{c,i} = 50 - 25 = 25^\circ\text{C}$

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1 / \Delta T_2)}$$

$$\Delta T_{lm} = \frac{36.17 - 25}{\ln(36.17 / 25)}$$

$$\Delta T_{lm} = \frac{11.17}{\ln(1.4468)}$$

$$\Delta T_{lm} = \frac{11.17}{0.3692}$$

$$\Delta T_{lm} = 30.25^\circ\text{C}$$

Step 4: Calculate Heat Transfer Area (A)

$$Q = UA\Delta T_{lm}$$

$$A = \frac{Q}{U\Delta T_{lm}}$$

$$A = \frac{174,723}{300 \times 30.25}$$
$$A = \frac{174,723}{9075}$$
$$A = 19.25 \text{ m}^2$$

Final Answer:

The heat exchanger area required is **19.25 m²**
