GUJARAT TECHNOLOGICAL UNIVERSITY

BE - 5 SEMESTER – MID TERM – W26 – ANSWER BANK

Subject Name & Code:

DYNAMICS OF MACHINERY - BE05000301

CHAPTER 3: BALANCING OF ROTATING AND RECIPROCATING SYSTEMS

1) Why balancing of rotating and reciprocating masses is necessary? What are effects of unbalancing? (CO.2) [3 Marks]

Balancing is necessary to eliminate undesirable dynamic forces and couples.

Effects of unbalancing:

- **Vibrations** in machine structure
- **Excessive bearing loads** and wear
- **Fatigue failure** of components
- **Noise** and reduced machine life
- **Loss of power** and efficiency

2) Differentiate between static balancing and dynamic balancing system. State the 2 practical examples of each. (CO.2) [4 Marks]

Static Balancing	Dynamic Balancing
Masses lie in single plane	Masses lie in different planes
Only forces are balanced	Forces and couples both balanced
Satisfied when centre of gravity lies on axis of rotation	Satisfied when axis of rotation is principal inertia axis
Single correction plane needed	Two correction planes needed
Rotor can be balanced at rest	Rotor must be running

Examples:

- **Static:** Disc wheel, Pulley
- **Dynamic:** Crankshaft, Turbine rotor

3) Write the short note on primary and secondary balancing. (CO.2) [3 Marks]

Primary Balancing:

- Balances primary forces occurring at **crank speed** (ω)
- Primary force = $m\omega^2 r \cos\theta$
- Completely balanced by counterweights

Secondary Balancing:

- Balances secondary forces occurring at **twice crank speed** (2ω)
- Secondary force = $(m\omega^2 r/n) \cos 2\theta$

- Cannot be completely balanced due to complexity
- Partial balancing reduces secondary effects

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4) Explain briefly the balancing of several masses in same plane. (CO.2) [4 Marks]
Analytical method:

- Resolve each centrifugal force into horizontal and vertical components
- **Resultant force** = $\sqrt{(\Sigma H^2 + \Sigma V^2)}$
- Where $\Sigma H = \Sigma mr\omega^2 \cos\theta$, $\Sigma V = \Sigma mr\omega^2 \sin\theta$
- **Balancing force** = Equal and opposite to resultant

Condition for complete balance:

- $\Sigma mr \cos\theta = 0$ (Horizontal balance)
- $\Sigma mr \sin\theta = 0$ (Vertical balance)

Procedure:

1. Draw space diagram with mass positions
2. Resolve forces in X and Y directions
3. Calculate resultant
4. Place balancing mass at 180° to resultant

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5) Justify that reciprocating masses are called as partially balanced. (CO.2) [3 Marks]
Reciprocating masses are **partially balanced** because:

- Primary force ($m\omega^2 r \cos\theta$) can be **completely balanced**
- Secondary force ($m\omega^2 r/n \cos 2\theta$) **cannot be completely balanced**
- Complete balancing would require **complex mechanisms**
- Attempting complete balance introduces **new unbalanced forces**
- Only **portion** of reciprocating mass is balanced by counterweights
- Remaining unbalanced force causes **vibrations**

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6) Explain the term hammer blow and how to control it. (CO.2) [3 Marks]
Hammer Blow: Maximum vertical unbalanced force acting on the rail due to the counterweight in a locomotive.
Formula: Hammer blow = $m\omega^2 r$ (for 2-cylinder locomotive)
How to control it:

- Use **partial balancing** of reciprocating masses
- Use **multiple cylinders** to distribute forces
- Use **counterweights** with proper proportion
- Use **balancing of locomotive** with suitable mass proportions

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7) Explain the following terms using neat sketches and equations: (CO.2) [7 Marks]
1. Variation of Tractive Force
Tractive Force: Horizontal component of unbalanced force along the line of stroke.
Equation:
$F_T = (1\text{-}c)\, m\omega^2 r\, [\cos\theta - \sin\theta]$ (for two-cylinder locomotive)
Variation:

- Maximum when $\cos\theta = \sin\theta \rightarrow \theta = 45°, 225°$
- Maximum tractive force = $\pm(1\text{-}c)m\omega^2 r\sqrt{2}$
- Varies sinusoidally during each revolution

Effects: Causes variation in driving wheel adhesion and slip.

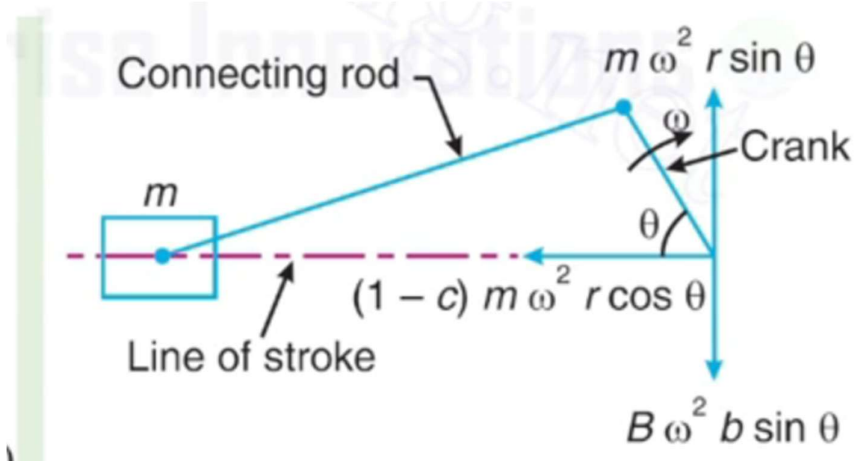


Fig. 22.4. Variation of tractive force.

2. Swaying Couple

Swaying Couple: Couple produced by unbalanced forces acting in opposite directions on two rails.

Equation:

$C_S = (1-c)m\omega^2 r \times (a/2)\,[\cos\theta + \sin\theta]$

Where a = distance between rails

Variation:

- Maximum when $\cos\theta + \sin\theta$ is maximum $\rightarrow \theta = 45°$
- Maximum couple = $(1-c)m\omega^2 r \times a/\sqrt{2}$

Effects: Causes swaying motion of locomotive.

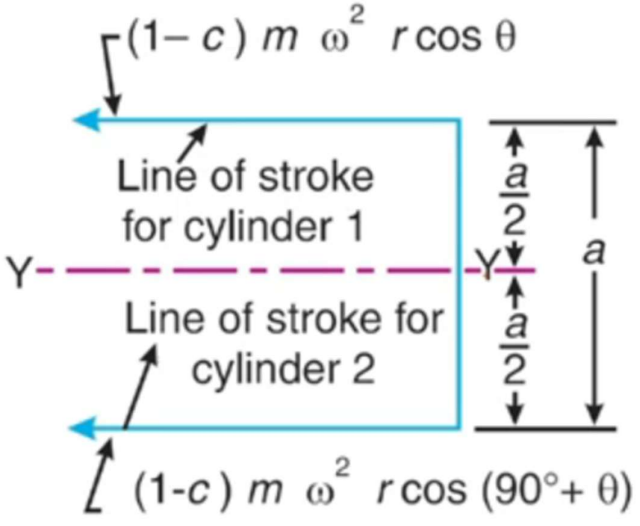


Fig. 22.5. Swaying couple.

CHAPTER 4: GYROSCOPE

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1) What do you understand by gyroscopic couple? Derive a formula for its magnitude. (CO.3) [4 Marks]

Gyroscopic Couple: The couple/torque required to change the plane of rotation of a spinning rotor.

Derivation:

Given:

- I = Mass moment of inertia
- ω = Angular velocity of spin
- ω_p = Angular velocity of precession

Formula:

$C = I \times \omega \times \omega_p$

Direction: Right-hand rule for torque

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2) Explain the following terms with neat sketch: (CO.3) [4 Marks]

i) Active Gyroscopic Couple

Active Gyroscopic Couple: Couple applied by the rotor on the frame/support.

- Opposes the change in direction of spin axis
- Acts on the supporting frame
- Direction: Opposite to precession

ii) Reactive Gyroscopic Couple

Reactive Gyroscopic Couple: Couple applied by the frame on the rotor.

- Causes the rotor to precess
- Equal and opposite to active couple
- Direction: Same as precession

Relation: Active Couple = - Reactive Couple



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3) Explain the following terms with neat sketch: (CO.3) [3 Marks]

i) Axis of Spin

- Axis about which the rotor/disc rotates
- Fixed in direction in space

ii) Angular Velocity of Spin (ω)

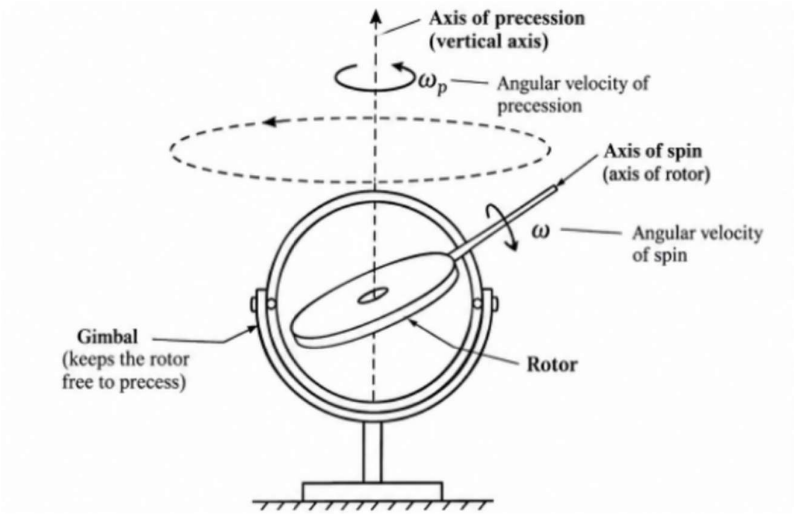
- Rate of rotation about axis of spin
- Units: rad/s

iii) Axis of Precession

- Axis about which the spin axis rotates
- Direction perpendicular to spin axis

iv) Angular Velocity of Precession (ω_p)

- Rate of change of direction of spin axis
- Units: rad/s



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4) Explain effect of gyroscopic couple on Ship during Pitching. (CO.3) [4 Marks]
Pitching: Oscillatory motion of ship about transverse axis.
Effects:
During bow down or up:

- Gyroscopic couple acts about vertical axis
- Tends to turn ship to right or left

During pitching:

- Angular velocity of pitch = ω_p
- Gyroscopic couple = I × ω × ω_p

Direction:

- When bow rises: Ship turns to one side
- When bow falls: Ship turns to opposite side

Conclusion: Gyroscopic effects cause **yawing** of ship during pitching.

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5) Explain the function of flywheel and how it is different from governor. (CO.3) [3 Marks]

Flywheel	**Governor**
Controls speed variation during each cycle	Controls mean speed over long periods
Stores energy when supply exceeds demand	Adjusts fuel supply to control speed
Acts during each revolution	Acts when load changes
Passive device	Active controlling device
Reduces fluctuations of speed	Maintains constant mean speed

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6) Explain effect of gyroscopic couple on aeroplane. (CO.3) [4 Marks]
When aeroplane takes left turn:
Given:

- Spin axis: Along propeller shaft
- Precession axis: Vertical (turning)

Effects:

- Gyroscopic couple acts about **pitch axis**
- Causes **nose to rise or dip**
- Direction depends on propeller rotation

For clockwise propeller (viewed from pilot):

- Turning left → Nose rises
- Turning right → Nose dips

Effects on control:

- Requires additional elevator control
- Affects aircraft stability
- Influences takeoff/landing

CHAPTER 5: FREE VIBRATIONS AND DAMPED FREE VIBRATIONS

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1) Define the following terms: (CO.4) [3 Marks]

1. Degrees of Freedom: Number of independent coordinates required to specify the configuration of a vibrating system.
2. Periodic Vibration: Motion that repeats itself after equal intervals of time.
3. Cycle: One complete sequence of periodic vibration (motion from one position back to same position with same velocity).
4. Frequency: Number of cycles completed per unit time. Units: Hz or cycles/s.
5. Time Period: Time required to complete one cycle. $T = 1/f$.
6. Stiffness of Spring: Force required to produce unit deflection. $k = F/x$.
7. Damped Vibration: Vibration with decreasing amplitude due to energy dissipation (friction/damping).
8. Equivalent spring stiffness: Combined stiffness of multiple springs in a system.

Configuration	Equivalent Stiffness
Series	$1/k_{eq} = 1/k_1 + 1/k_2$
Parallel	$k_{eq} = k_1 + k_2$

9. Resonance: When natural frequency equals exciting frequency, resulting in maximum amplitude.
10. Steady state vibration: Vibration after transients die out. **Transient vibration:** Vibration during initial period before steady state.
11. Undamped Free Vibration: Vibration of a system without damping after initial disturbance.
12. Equivalent Damper in Series: $1/C_{eq} = 1/C_1 + 1/C_2$
13. Critical Damping Coefficient: Minimum damping that prevents oscillation. $C_c = 2m\omega_n$
14. Damping ratio: $\zeta = C/C_c$
15. Period: Time for one complete cycle. **Cycle:** Complete oscillation.
16. Logarithmic decrement: Natural logarithm of ratio of successive amplitudes. $\delta = \ln(x_1/x_2)$

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2) State main cause of vibration and application of vibration. (CO.4) [3 Marks]

Causes of Vibration:

- Unbalanced forces in machines
- External forces (wind, earthquakes)
- Rotating/reciprocating parts
- Manufacturing defects
- Friction

Applications:

- **Vibration testing** for quality control
- **Screening** and **sorting** (sieves)
- **Ultrasonic cleaning** and **drilling**
- **Medical imaging** (ultrasound)
- **Material compaction** (concrete vibrators)

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3) Classify types of vibration system. (CO.4) [3 Marks]

Classification:

1. Based on Energy Source:
- Free vibration (no external force)
- Forced vibration (external force)

2. Based on Damping:
- Undamped vibration
- Damped vibration
- Overdamped, Critically damped, Underdamped

3. Based on DOF:
- Single DOF
- Multi DOF

4. Based on Motion Type:
- Longitudinal
- Transverse/Lateral
- Torsional

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4) Derive an expression for natural frequency of cantilever beam subjected to point load at the end. (CO.4) [4 Marks]

Given: Cantilever beam with end mass M and point load P

Deflection at free end:

$\delta = PL^3/3EI$

Stiffness of beam:

$k = P/\delta = 3EI/L^3$

Natural frequency:

$\omega_n = \sqrt{(k/m)} = \sqrt{(3EI/(mL^3))}$

In Hz:

$f_n = (1/2\pi)\sqrt{(3EI/(mL^3))}$

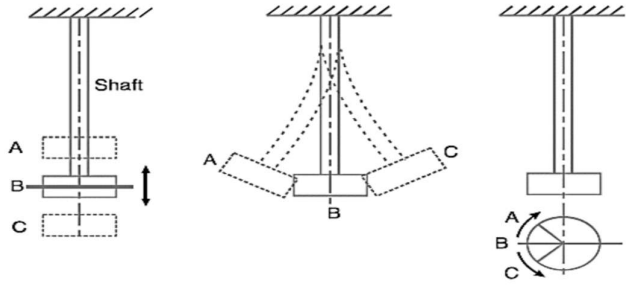
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5) Explain the following terms with neat sketches: (CO.4) [3 Marks]

1. Longitudinal Vibration:
- Particles move parallel to axis of shaft
- Stretching and compressing motion
- Example: Spring-mass system

2. Lateral Vibration:
- Particles move perpendicular to axis
- Transverse bending motion
- Example: Cantilever beam deflection

3. Torsional Vibration:
- Particles twist about axis
- Angular oscillation
- Example: Disc on shaft



B = Mean position ; *A* and *C* = Extreme positions.

(a) Longitudinal Vibrations (b) Transverse Vibrations (c) Torsional Vibrations

Fig 5.1 Types of free vibrations

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6) Derive equation of motion for free vibration with viscous damping having single degree of freedom translational vibration system by Newton's method or D'Alembert method. (CO.4) [7 Marks]

Equation of motion using Newton's Method:

Given: Mass m, Spring k, Damper c

Free Body Diagram forces:

- Spring force: kx (restoring)
- Damping force: cẋ (opposing motion)
- Inertia force: mẍ (D'Alembert)

Summing forces:

mẍ + cẋ + kx = 0

D'Alembert's Method:

mẍ + cẋ + kx = 0

Solution:

x = Ae^(λt)

Characteristic equation:

mλ² + cλ + k = 0

Roots:

λ = [-c ± √(c² - 4mk)]/2m

Natural frequency:

ω_n = √(k/m)

Damping ratio:

ζ = c/(2√(mk)) = c/c_c

Final equation:

mẍ + cẋ + kx = 0

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7) In viscous damping, proof that the damping force is proportional to the velocity of the vibrating body. (CO.4) [3 Marks]

Viscous Damping:

- Damping force acts opposite to velocity
- For a flat plate moving through fluid

Fluid shear stress: τ = μ(du/dy)

Shear force: F = μA(du/dy)

For small gap: du/dy = v/h

Damping force: F = μA(v/h)

Therefore:

F ∝ v

F = cv

Where c = μA/h = damping coefficient

Conclusion: Damping force is **directly proportional** to velocity for viscous damping.

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8) Draw & explain the response curves for Overdamped, Underdamped, and Critical damped systems. (CO.4) [3 Marks]

Response curves:

1. Underdamped (ζ < 1):

- Oscillatory response
- Amplitude decreases exponentially
- x = Xe^(-ζω_n t) sin(ω_d t + φ)

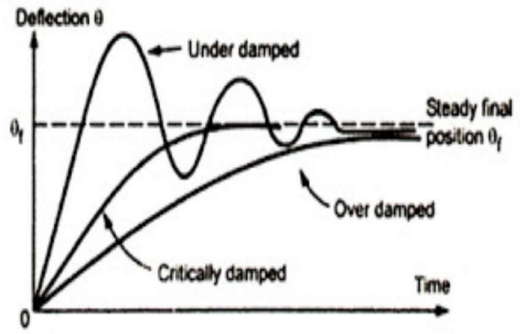
2. Critically Damped (ζ = 1):

- No oscillation
- Returns to equilibrium in shortest time
- x = (A + Bt)e^(-ω_n t)

3. Overdamped (ζ > 1):

- No oscillation
- Returns slowly to equilibrium
- x = Ae^(-λ₁t) + Be^(-λ₂t)

Comparison: Critical damping gives fastest return to rest without oscillation.



Damping System

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9) Explain logarithmic decrement. Derive the equation to determine logarithmic decrement. (CO.4) [4 Marks]

Logarithmic decrement: Natural logarithm of ratio of two successive amplitudes.

Derivation:

Given: $x_1 = Xe^{(-\zeta\omega_n t_1)}$, $x_2 = Xe^{(-\zeta\omega_n t_2)}$

Ratio: $x_1/x_2 = e^{(\zeta\omega_n (t_2 - t_1))}$

But $t_2 - t_1 = T_d$ (damped period)

Therefore: $x_1/x_2 = e^{(\zeta\omega_n T_d)}$

Taking log: $\ln(x_1/x_2) = \zeta\omega_n T_d$

Since $T_d = 2\pi/\omega_d = 2\pi/\omega_n\sqrt{(1-\zeta^2)}$:

Final equation:

$\delta = \ln(x_1/x_2) = 2\pi\zeta/\sqrt{(1-\zeta^2)}$

For small damping: $\delta \approx 2\pi\zeta$

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10) Derive an expression for length of torsionally equivalent shaft system. (CO.4) [4 Marks]

For Shaft in Series:

Given: n shafts with lengths $L_1, L_2, ..., L_n$ and polar moments $J_1, J_2, ..., J_n$

Stiffness relation:

$1/k_t = 1/k_1 + 1/k_2 + ... + 1/k_n$

Where: $k_i = GJ_i/L_i$

Therefore:

$L_{eq}/(GJ_{eq}) = L_1/(GJ_1) + L_2/(GJ_2) + ... + L_n/(GJ_n)$

For same material:

$L_{eq}/J_{eq} = L_1/J_1 + L_2/J_2 + ... + L_n/J_n$

If $J_{eq} = J_{base}$ (reference):

$L_{eq} = L_1(J_{base}/J_1) + L_2(J_{base}/J_2) + ... + L_n(J_{base}/J_n)$

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11) Derive an expression for natural frequency of torsional vibration of a disc connected

at the end of the shaft. (CO.4) [4 Marks]
Given:

- Shaft length L, diameter d
- Modulus of rigidity G
- Disc moment of inertia I

Torsional stiffness:
k_t = GJ/L = G(πd⁴/32)/L
Equation of motion:
Iθ̈ + k_t θ = 0
Natural frequency:
ω_n = √(k_t/I)
Substituting:
ω_n = √(GJ/(IL))
In terms of shaft diameter:
ω_n = √(Gπd⁴/(32IL))
Final expression:
ω_n = √(GJ/(IL)) rad/s

Numerical:

1.Problem Statement
The cockpit of a firetruck is located at the end of a telescoping boom, as shown in Figure. The cockpit, along with the fireman, weighs **2000 N**. Find the cockpit's natural frequency of vibration in the vertical direction.
Given Data:

- Young's modulus: **$E = 2.1 \times 10^{11}$ N/m²**
- Lengths: **$l_1 = l_2 = l_3 = 3$ m**
- Cross-sectional areas:
  - **$A_1 = 20$ cm²** = 20×10^{-4} m²
  - **$A_2 = 10$ cm²** = 10×10^{-4} m²
  - **$A_3 = 5$ cm²** = 5×10^{-4} m²
- Weight: **W = 2000 N**
- Angle of boom: **45°**

Solution Approach
To determine the natural frequency, we find the **equivalent stiffness** of the boom in the vertical direction and use a **single-degree-of-freedom** idealization.
Assumptions:

1. Mass of telescoping boom is **negligible**
2. Boom deforms only in the **axial direction** (no bending)
3. Force induced at any cross-section is equal to the axial load at the end

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Step 1: Calculate Axial Stiffness of Each Segment
The axial stiffness of each segment is given by:

$$k_{bi} = \frac{A_i E}{l_i}$$

For Segment 1:

$$k_{b1} = \frac{(20 \times 10^{-4})(2.1 \times 10^{11})}{3}$$
$$k_{b1} = \frac{4.2 \times 10^{8}}{3}$$
$$k_{b1} = 14 \times 10^{7} \text{ N/m}$$

For Segment 2:

$$k_{b2} = \frac{(10 \times 10^{-4})(2.1 \times 10^{11})}{3}$$
$$k_{b2} = \frac{2.1 \times 10^{8}}{3}$$
$$k_{b2} = 7 \times 10^{7} \text{ N/m}$$

For Segment 3:

$$k_{b3} = \frac{(5 \times 10^{-4})(2.1 \times 10^{11})}{3}$$
$$k_{b3} = \frac{1.05 \times 10^{8}}{3}$$
$$k_{b3} = 3.5 \times 10^{7} \text{ N/m}$$

Step 2: Calculate Equivalent Axial Stiffness (Series Combination)

For springs in series:

$$\frac{1}{k_b} = \frac{1}{k_{b1}} + \frac{1}{k_{b2}} + \frac{1}{k_{b3}}$$

$$\frac{1}{k_b} = \frac{1}{10^7}\left(\frac{1}{14} + \frac{1}{7} + \frac{1}{3.5}\right)$$

Now, $\frac{1}{14} = 0.0714$, $\frac{1}{7} = 0.1429$, $\frac{1}{3.5} = 0.2857$

$$\frac{1}{k_b} = \frac{1}{10^7}(0.0714 + 0.1429 + 0.2857)$$
$$\frac{1}{k_b} = \frac{1}{10^7}(0.5)$$
$$\frac{1}{k_b} = \frac{1}{2 \times 10^7}$$
$$k_b = 2 \times 10^7 \text{ N/m}$$

Step 3: Find Vertical Stiffness (Considering 45° Angle)

The stiffness of the telescoping boom in the **vertical direction** is:

$$k = k_b \cos^2 45^\circ$$
$$k = (2 \times 10^7) \times (0.707)^2$$

Since $\cos 45^\circ = \frac{1}{\sqrt{2}}$, $\cos^2 45^\circ = \frac{1}{2}$

$$k = 2 \times 10^7 \times \frac{1}{2}$$
$$k = 1 \times 10^7 \text{ N/m}$$

Step 4: Calculate Mass

$$m = \frac{W}{g} = \frac{2000}{9.81}$$
$$m = 203.87 \text{ kg}$$

Step 5: Calculate Natural Frequency

$$\omega_n = \sqrt{\frac{k}{m}}$$
$$\omega_n = \sqrt{\frac{10^7}{203.87}}$$
$$\omega_n = \sqrt{49{,}050}$$
$$\omega_n = 221.47 \text{ rad/s}$$

2.Problem Statement

Determine the natural frequency of the pulley system shown in Figure. Assume the pulleys to be **frictionless** and of **negligible mass**.
(Dec-20) | CO.4

Given Data

- Spring stiffness: $\mathbf{k_1}$ and $\mathbf{k_2}$
- Mass: **m**
- Pulleys: Frictionless and massless

Solution Approach

To determine the natural frequency, we find the **equivalent stiffness** of the system and solve it as a **single-degree-of-freedom** problem.
Key Assumptions:

1. Pulleys are frictionless and massless
2. Tension in the rope is constant throughout
3. Rope is inextensible

Step 1: Determine Tension in the Rope

Since the pulleys are frictionless and massless, the **tension in the rope is constant** and is equal to the weight of the mass:

$$T = W = mg$$

Step 2: Analyze Forces on Pulleys
For Pulley 1:

- Upward force on pulley 1 from spring: $2T$
- Therefore, force on spring k_1 is $2W$

For Pulley 2:

- Downward force on pulley 2 from spring: $2T$
- Therefore, force on spring k_2 is $2W$

Step 3: Determine Displacements
Displacement of Pulley 1 Center (Point A):

$$x_1 = \frac{2W}{k_1}$$

Displacement of Pulley 2 Center (Point B):

$$x_2 = \frac{2W}{k_2}$$

Step 4: Determine Displacement of Mass (Point O)

Rope Geometry:

- One rope segment is attached to mass O at point O
- It passes over pulley 1, then pulley 2, and is fixed at the end

From the rope kinematics:

$$X = 2x_1 + 2x_2$$

Where:

- X = displacement of mass m
- x_1 = displacement of pulley 1 center
- x_2 = displacement of pulley 2 center

Substituting values:

$$X = 2\left(\frac{2W}{k_1}\right) + 2\left(\frac{2W}{k_2}\right)$$
$$X = \frac{4W}{k_1} + \frac{4W}{k_2}$$
$$X = 4W\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

Step 5: Find Equivalent Stiffness

The equivalent stiffness is defined as:

$$k_{eq} = \frac{\text{Force}}{\text{Displacement}} = \frac{W}{X}$$

Substituting:

$$k_{eq} = \frac{W}{4W\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}$$
$$k_{eq} = \frac{1}{4\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}$$
$$k_{eq} = \frac{1}{4} \times \frac{k_1 k_2}{k_1 + k_2}$$
$$\boxed{k_{eq} = \frac{k_1 k_2}{4(k_1 + k_2)}}$$

Step 6: Derive Equation of Motion

When mass m is displaced from static equilibrium position by x:

$$m\ddot{x} + k_{eq}x = 0$$

Step 7: Natural Frequency (Angular)

$$\omega_n = \sqrt{\frac{k_{eq}}{m}}$$
$$\omega_n = \sqrt{\frac{k_1 k_2}{4m(k_1 + k_2)}}$$
$$\boxed{\omega_n = \frac{1}{2}\sqrt{\frac{k_1 k_2}{m(k_1 + k_2)}} \text{ rad/s}}$$

Step 8: Natural Frequency (Hz)

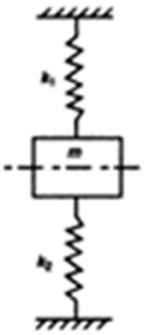
$$f_n = \frac{\omega_n}{2\pi}$$

$$\boxed{f_n = \frac{1}{4\pi}\sqrt{\frac{k_1 k_2}{m(k_1 + k_2)}}\ \text{Hz}}$$

3.Problem Statement

Evaluate the natural frequency of a vibratory system shown in figure. The mass of block is **m** and stiffness of springs are $\mathbf{k_1}$ and $\mathbf{k_2}$.

(Winter-2024) | CO.4



Case 2: Springs in Series

When springs are connected **in series** (one after another):

Step 1: Equivalent Stiffness

For series springs:

$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$k_{eq} = \frac{k_1 k_2}{k_1 + k_2}$$

Step 2: Natural Frequency Formula

$$\omega_n = \sqrt{\frac{k_{eq}}{m}}$$

Step 3: Final Expression

$$\boxed{\omega_n = \sqrt{\frac{k_1 k_2}{m(k_1 + k_2)}}}$$

Step 4: In Hz

$$\boxed{f_n = \frac{1}{2\pi}\sqrt{\frac{k_1 k_2}{m(k_1 + k_2)}}}$$

4. Problem Statement

A vibrating system is defined by the following parameters:

- **m = 3 kg**
- **k = 100 N/m**
- **c = 3 N.s/m**

Determine:

1. Damping factor (ζ)
2. Natural frequency of damped vibration (ωd)
3. Logarithmic decrement (δ)
4. Ratio of two consecutive amplitudes (x_1/x_2)
5. Number of cycles after which the original amplitude is reduced to 20%

(Winter-2022) | CO.4

Step 1: Calculate Natural Frequency (Undamped)

$$\omega_n = \sqrt{\frac{k}{m}}$$
$$\omega_n = \sqrt{\frac{100}{3}}$$
$$\omega_n = \sqrt{33.33}$$
$$\omega_n = 5.77 \text{ rad/s}$$

Step 2: Calculate Critical Damping Coefficient

$$c_c = 2\sqrt{km}$$
$$c_c = 2\sqrt{100 \times 3}$$
$$c_c = 2\sqrt{300}$$
$$c_c = 2 \times 17.32$$
$$c_c = 34.64 \text{ N.s/m}$$

(i) Damping Factor (ζ)

$$\zeta = \frac{c}{c_c}$$
$$\zeta = \frac{3}{34.64}$$
$$\boxed{\zeta = 0.0866}$$

(ii) Natural Frequency of Damped Vibration (ωd)

$$\omega_d = \omega_n\sqrt{1 - \zeta^2}$$
$$\omega_d = 5.77\sqrt{1 - (0.0866)^2}$$
$$\omega_d = 5.77\sqrt{1 - 0.0075}$$
$$\omega_d = 5.77\sqrt{0.9925}$$
$$\omega_d = 5.77 \times 0.9962$$
$$\boxed{\omega_d = 5.75 \text{ rad/s}}$$

(iii) Logarithmic Decrement (δ)

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$
$$\delta = \frac{2\pi \times 0.0866}{\sqrt{1-0.0075}}$$
$$\delta = \frac{0.544}{0.9962}$$
$$\boxed{\delta = 0.546}$$

(iv) Ratio of Two Consecutive Amplitudes (x_1/x_2)

$$\frac{x_1}{x_2} = e^{\delta}$$
$$\frac{x_1}{x_2} = e^{0.546}$$
$$\boxed{\frac{x_1}{x_2} = 1.726}$$

(v) Number of Cycles for Amplitude to Reduce to 20%

Original amplitude reduced to 20% means:

$$\frac{x_n}{x_0} = 0.20$$

For n cycles:

$$\frac{x_n}{x_0} = e^{-n}$$
$$0.20 = e^{-n\times 0.546}$$

Taking natural log on both sides:

$$\ln(0.20) = -n \times 0.546$$
$$-1.609 = -n \times 0.546$$
$$n = \frac{1.609}{0.546}$$
$$\boxed{n = 2.95 \approx 3 \text{ cycles}}$$

5. problem Statement

A spring-mass system consists of:

- **Spring stiffness: k = 350 N/m**
- **Mass: m = 0.35 kg**
- **Initial displacement: x_0 = 20 mm** (beyond equilibrium)
- **Damping coefficient: c = 14 Ns/m**

Calculate:

1. Critical damping coefficient (cc)
2. Damped natural frequency (ωd)
3. Logarithmic decrement (δ)

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Step 1: Calculate Critical Damping Coefficient (cc)

$$c_c = 2\sqrt{km}$$
$$c_c = 2\sqrt{350 \times 0.35}$$
$$c_c = 2\sqrt{122.5}$$
$$c_c = 2 \times 11.068$$

$$\boxed{c_c = 22.14\ \text{Ns/m}}$$

Step 2: Calculate Natural Frequency (Undamped)

$$\omega_n = \sqrt{\frac{k}{m}}$$
$$\omega_n = \sqrt{\frac{350}{0.35}}$$
$$\omega_n = \sqrt{1000}$$
$$\omega_n = 31.62\ \text{rad/s}$$

Step 3: Calculate Damping Factor (ζ)

$$\zeta = \frac{c}{c_c}$$
$$\zeta = \frac{14}{22.14}$$
$$\zeta = 0.632$$

Step 4: Calculate Damped Natural Frequency (ωd)

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$
$$\omega_d = 31.62\sqrt{1-(0.632)^2}$$
$$\omega_d = 31.62\sqrt{1-0.399}$$
$$\omega_d = 31.62\sqrt{0.601}$$
$$\omega_d = 31.62 \times 0.775$$
$$\boxed{\omega_d = 24.51\ \text{rad/s}}$$

Step 5: Calculate Logarithmic Decrement (δ)

$$\delta = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}}$$
$$\delta = \frac{2\pi \times 0.632}{\sqrt{1-0.399}}$$
$$\delta = \frac{3.971}{\sqrt{0.601}}$$
$$\delta = \frac{3.971}{0.775}$$
$$\boxed{\delta = 5.12}$$

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