

Subject Name & Code:
NMEE (BE03000191)

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Assignment – 2

CO-2: Numerical Solution of Algebraic and Transcendental Equation

1. Bisection Method

Formula:

$$x_m = \frac{a + b}{2}$$

where a, b are the current interval endpoints and $f(a)f(b) < 0$.

Q1. $f(x) = x^3 - x - 2$, interval $[1, 2]$

Iteration	a	b	x_m	$f(x_m)$
1	1.000000	2.000000	1.500000	-0.125000
2	1.500000	2.000000	1.750000	1.609375
3	1.500000	1.750000	1.625000	0.666016
4	1.500000	1.625000	1.562500	0.252197
5	1.500000	1.562500	1.531250	0.058838
6	1.500000	1.531250	1.515625	-0.033600
7	1.515625	1.531250	1.523438	0.012600
8	1.515625	1.523438	1.519531	-0.010500

Iteration	a	b	x_m	$f(x_m)$
9	1.519531	1.523438	1.521484	0.001100
10	1.519531	1.521484	1.520508	-0.004700

After sufficient iterations,

$$x \approx 1.521380$$

Q2. $f(x) = \cos x - x$, interval $[0, 1]$

Iteration	a	b	x_m	$f(x_m)$
1	0.000000	1.000000	0.500000	0.377583
2	0.500000	1.000000	0.750000	-0.018311
3	0.500000	0.750000	0.625000	0.185963
4	0.625000	0.750000	0.687500	0.085335
5	0.687500	0.750000	0.718750	0.033000
6	0.718750	0.750000	0.734375	0.007200
7	0.734375	0.750000	0.742188	-0.005500
8	0.734375	0.742188	0.738281	0.000800

Hence,

$$x \approx 0.739085$$

Q3. $f(x) = x^2 - 4$, interval $[1, 3]$

$$f(1) = -3, f(3) = 5$$

First midpoint:

$$x_m = \frac{1+3}{2} = 2$$

$$f(2) = 2^2 - 4 = 0$$

Therefore the exact root is

$$x = 2.000000$$

2. False Position Method / Regula Falsi

Formula:

$$x_r = a - \frac{f(a)(b-a)}{f(b)-f(a)}$$

or equivalently,

$$x_r = b - \frac{f(b)(b-a)}{f(b)-f(a)}$$

Q4. $f(x) = x^3 - x - 2$, interval $[1, 2]$

Iteration	a	b	x_r	$f(x_r)$
1	1.000000	2.000000	1.333333	-0.962963
2	1.333333	2.000000	1.462687	-0.334000
3	1.462687	2.000000	1.504100	-0.102000
4	1.504100	2.000000	1.516400	-0.029000
5	1.516400	2.000000	1.519900	-0.008000
6	1.519900	2.000000	1.520900	-0.002000

Final root:

$$x \approx 1.521380$$

Q5. $f(x) = e^x - 3x$, interval $[0, 1]$

$$f(0) = 1, f(1) = e - 3 = -0.281718$$

Iteration	a	b	x_r	$f(x_r)$
1	0.000000	1.000000	0.780202	-0.158600
2	0.000000	0.780202	0.673400	-0.059400
3	0.000000	0.673400	0.635700	-0.019000
4	0.000000	0.635700	0.623900	-0.006000
5	0.000000	0.623900	0.620200	-0.002000
6	0.000000	0.620200	0.619000	-0.000600

Final root:

$$x \approx 0.619061$$

Q6. $f(x) = \ln x + x^2 - 3$, interval $[1, 2]$

$$f(1) = -2, f(2) = \ln 2 + 4 - 3 = 1.693147$$

Iteration	a	b	x_r	$f(x_r)$
1	1.000000	2.000000	1.541600	-0.190700
2	1.541600	2.000000	1.598700	0.025000
3	1.541600	1.598700	1.592100	-0.000500
4	1.592100	1.598700	1.592143	0.000000

$$x \approx 1.592143$$

Note: This corrects the apparent typo in the given answer key.

3. Secant Method

Formula:

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})}$$

Q7. $f(x) = x^3 - x - 2$, $x_0 = 1$, $x_1 = 2$

n	x_n	$f(x_n)$
0	1.000000	-2.000000
1	2.000000	4.000000
2	1.333333	-0.962963
3	1.462687	-0.334000
4	1.531400	0.061000
5	1.520800	-0.002000
6	1.521380	0.000000

$$x \approx 1.521380$$

Q8. $f(x) = \tan x - x$, $x_0 = 4.5$, $x_1 = 4.7$

Using the secant formula repeatedly gives convergence near the first positive root after π :

$$x \approx 4.493409$$

Q9. $f(x) = xe^x - 1$, $x_0 = 0$, $x_1 = 1$

$$f(0) = -1, f(1) = e - 1 = 1.718282$$

$$x_2 = 1 - \frac{1.718282(1 - 0)}{1.718282 - (-1)} = 0.367879$$

$$x_3 \approx 0.503400, x_4 \approx 0.549000$$

After further iterations:

$$\boxed{x \approx 0.567143}$$

4. Newton-Raphson Method

Formula:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Q10. $f(x) = x^3 - x - 2, x_0 = 1.5$

$$f'(x) = 3x^2 - 1$$

$$x_1 = 1.5 - \frac{1.5^3 - 1.5 - 2}{3(1.5)^2 - 1} = 1.5 - \frac{-0.125}{5.75} = 1.521739$$

$$x_2 \approx 1.521380$$

$$\boxed{x \approx 1.521380}$$

Q11. $f(x) = \cos x - x, x_0 = 0.5$

$$f'(x) = -\sin x - 1$$

$$x_1 = 0.5 - \frac{\cos 0.5 - 0.5}{-\sin 0.5 - 1} \approx 0.755200$$

$$x_2 \approx 0.739100, x_3 \approx 0.739085$$

$$\boxed{x \approx 0.739085}$$

Q12. $f(x) = e^x - 3x, x_0 = 0.5$

$$f'(x) = e^x - 3$$

$$x_1 = 0.5 - \frac{e^{0.5} - 1.5}{e^{0.5} - 3} \approx 0.610100$$

$$x_2 \approx 0.619100, x_3 \approx 0.619061$$

$$\boxed{x \approx 0.619061}$$

5. Fixed Point Iteration Method

Formula:

$$x_{n+1} = g(x_n)$$

Q13. $x^3 - x - 2 = 0$, rewritten as $x = \sqrt[3]{x+2}$

Take $x_0 = 1.5$.

n	x_n
0	1.500000
1	1.518294
2	1.520900
3	1.521300
4	1.521380

$$x \approx 1.521380$$

Q14. $x = \cos x$, $x_0 = 0.5$

n	x_n
0	0.500000
1	0.877583
2	0.639012
3	0.802686
4	0.694778
5	0.768196
6	0.719399
7	0.752356

n	x_n
8	0.730100
...	...

After sufficient iterations:

$$x \approx 0.739085$$

Q15. $x = \sqrt{3 - x}$, $x_0 = 1$

n	x_n
0	1.000000
1	1.414214
2	1.259280
3	1.319360
4	1.296400
5	1.305220
6	1.301840
7	1.303140
8	1.302640
9	1.302830
10	1.302750

$$x \approx 1.302776$$

