GUJARAT TECHNOLOGICAL UNIVERSITY

BE-5 SEMESTER – OLD PAPER – S22 TO S26 – ANSWER BANK

Subject Name & Code: Heat Transfer - 3151909

Chapter 1: Conduction & Critical Radius of Insulation

Repeated Questions:

1. General Heat Conduction Equation:

- **Q.** Derive the general heat conduction equation in Cartesian/Cylindrical coordinates. **(SUMMER 2022, 7 Marks; WINTER 2022, 7 Marks OR; WINTER 2023, 7 Marks OR; WINTER 2024, 7 Marks OR; SUMMER 2025, 7 Marks; SUMMER 2026, 7 Marks)**

Answer:

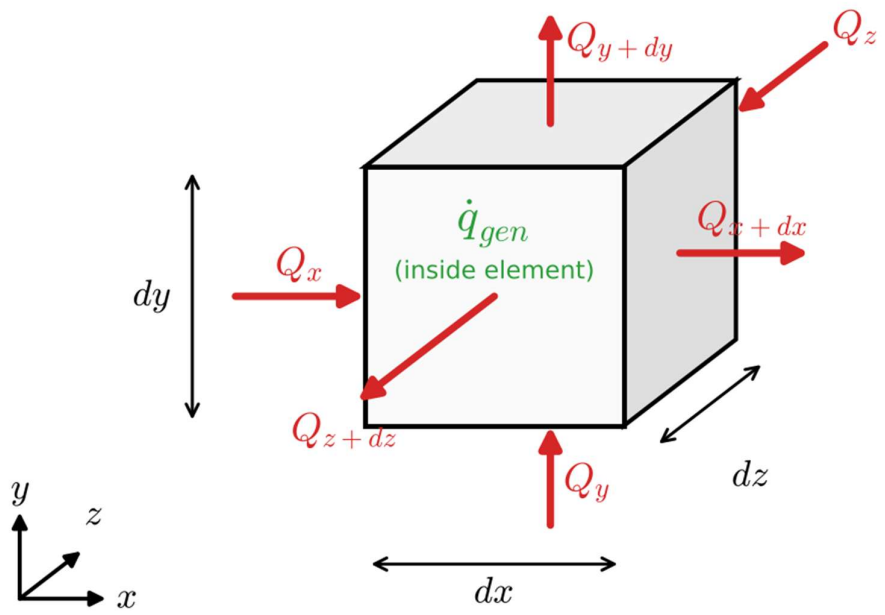


Fig. 1: Differential rectangular element dx dy dz with heat inflow, outflow and internal heat generation

- **Assumptions:** Homogeneous, isotropic material; constant thermal conductivity k; continuum; heat generation $\dot{q}_g$ per unit volume.

Steps:

1. Apply energy balance on the element:

$$q_x + q_y + q_z + \dot{q}_g\, dx\, dy\, dz = q_{x+dx} + q_{y+dy} + q_{z+dz} + \rho c_p\, dx\, dy\, dz\, \frac{\partial T}{\partial t}$$

2. By Fourier's law:

$$q_x = -k\, dy\, dz \frac{\partial T}{\partial x}, \quad q_y = -k\, dx\, dz \frac{\partial T}{\partial y}, \quad q_z = -k\, dx\, dy \frac{\partial T}{\partial z}$$

3. Use Taylor expansion:

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx$$

and similarly for y, z.

4. Substitute and divide by $dx\,dy\,dz$:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}_g=\rho c_p\frac{\partial T}{\partial t}$$

For constant k, with thermal diffusivity $\alpha = k/(\rho c_p)$:

$$\boxed{\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}_g}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

Cylindrical coordinates final result:

$$\boxed{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}_g}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

- **Q.** Deduce the general equation to Laplace, Poisson, and Fourier equations. **(WINTER 2022, 7 Marks; SUMMER 2024, 7 Marks; WINTER 2024, 7 Marks)**

Answer:

From general equation:

$$\nabla^2 T+\frac{\dot{q}_g}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Poisson equation — steady state with heat generation:

$$\frac{\partial T}{\partial t}=0,\ \dot{q}_g\neq 0$$

$$\boxed{\nabla^2 T=-\frac{\dot{q}_g}{k}}$$

Laplace equation — steady state, no heat generation:

$$\frac{\partial T}{\partial t}=0,\ \dot{q}_g=0$$

$$\boxed{\nabla^2 T=0}$$

Fourier equation — transient, no heat generation:

$$\dot{q}_g=0$$

$$\boxed{\nabla^2 T=\frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

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2. Critical Radius of Insulation:

- **Q.** Concept and Derivation of Critical Radius of Insulation. **(WINTER 2023, 3 Marks; SUMMER 2024, 4 Marks - for sphere; WINTER 2024, 3 Marks; SUMMER 2025, 3 Marks - for sphere; SUMMER 2026, 4 Marks OR - for cylinder)**

Answer:

- **Concept:** Adding insulation does not always reduce heat loss. For small pipe/cylinder radius, adding insulation increases outer area and may increase heat transfer until a critical radius is reached. Beyond r_c, further insulation reduces heat loss.

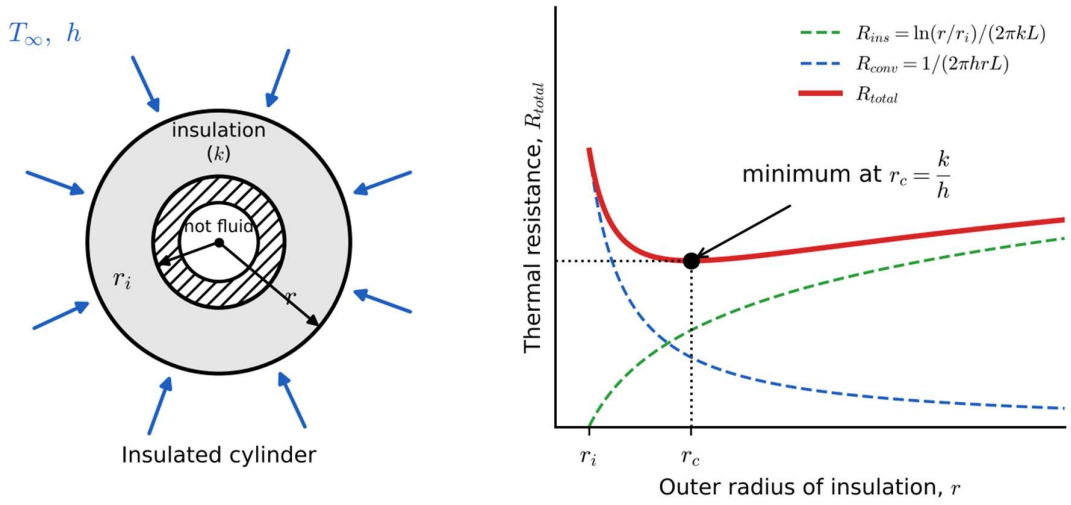


Fig. 2: Insulated cylinder (inner radius r_i, outer radius r) exposed to ambient (h), and R_{total} vs r with minimum at r_c

For Cylinder

Thermal resistance:

$$R_{\text{total}} = \frac{\ln(r/r_i)}{2\pi kL} + \frac{1}{h(2\pi rL)}$$

Differentiate w.r.t. r:

$$\frac{dR_{\text{total}}}{dr} = \frac{1}{2\pi kLr} - \frac{1}{2\pi hLr^2}$$

Set $\frac{dR}{dr} = 0$:

$$\frac{1}{kr} = \frac{1}{hr^2}$$

$$\boxed{r_c = \frac{k}{h}}$$

For Sphere

$$R_{\text{total}} = \frac{1}{4\pi k}\left(\frac{1}{r_i} - \frac{1}{r}\right) + \frac{1}{4\pi hr^2}$$

$$\frac{dR}{dr} = \frac{1}{4\pi kr^2} - \frac{1}{2\pi hr^3} = 0$$

$$\boxed{r_c = \frac{2k}{h}}$$

Conclusion:

- If $r_i < r_c$, adding insulation up to r_c increases heat transfer.
- If $r_i \geq r_c$, adding insulation always decreases heat transfer.

3. Composite Walls / Insulation Numericals:

- **Q.** Numericals on Composite Walls/Insulation (Application of electrical analogy). [Diagram required – refer to the original question paper of SUMMER 2022] **(SUMMER 2022; WINTER 2023, 7 Marks - Thermopane Window)**

Answer:

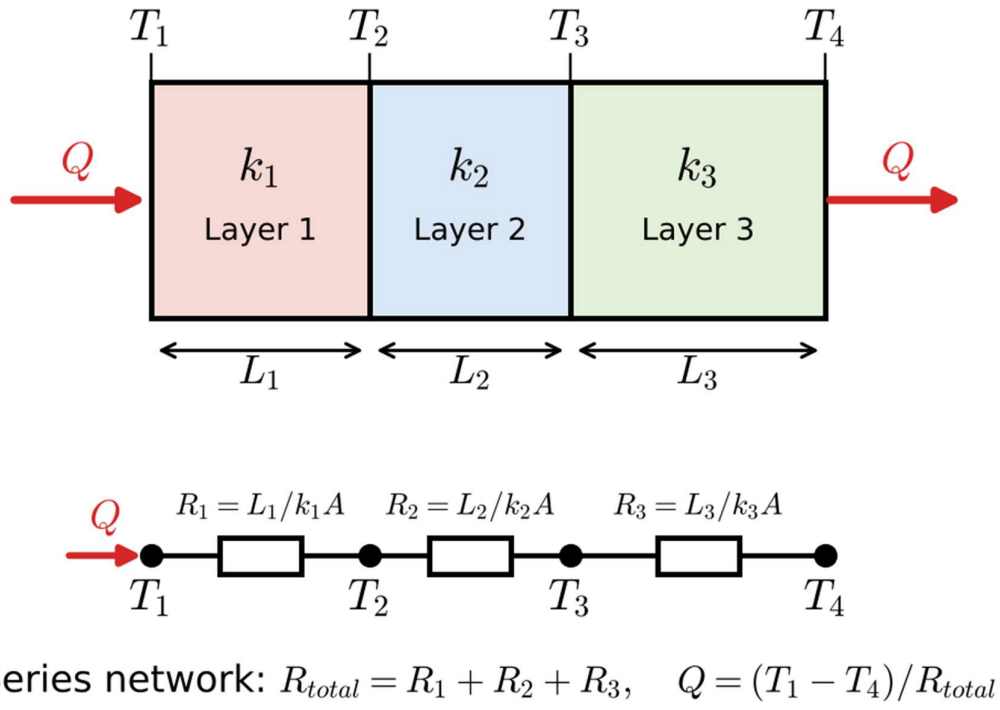


Fig. 3: Composite wall of three layers in series with its thermal resistance network

Electrical analogy:

$$Q = \frac{\Delta T}{R_{\text{total}}}$$

where

$$R_{\text{plane wall}} = \frac{L}{kA}, \quad R_{\text{cylinder}} = \frac{\ln(r_2/r_1)}{2\pi kL}, \quad R_{\text{conv}} = \frac{1}{hA}$$

For series composite wall:

$$Q = \frac{T_1 - T_n}{\frac{L_1}{k_1A} + \frac{L_2}{k_2A} + \cdots + \frac{L_n}{k_nA}}$$

Overall heat transfer coefficient:

$$U = \frac{1}{A\sum R}$$

Thermopane window method:

$$Q = \frac{\Delta T}{\frac{1}{h_iA} + \frac{L_g}{k_gA} + \frac{L_{air}}{k_{air}A} + \frac{L_g}{k_gA} + \frac{1}{h_oA}}$$

4. Thermal Diffusivity & Basic Definitions:

- **Q.** Define Thermal Diffusivity and explain its significance. **(WINTER 2023, 3 Marks; SUMMER 2024, 3 Marks; WINTER 2025, 3 Marks; SUMMER 2026, 3 Marks)**

Answer:

Thermal diffusivity:

$$\boxed{\alpha = \frac{k}{\rho c_p}}$$

Units: $\boxed{m^2/s}$

- **Significance:** It indicates how quickly a temperature disturbance diffuses through a material relative to its ability to store heat. Higher α means faster temperature response and quicker propagation of heat.
- **Thermal capacity:** Heat required to raise temperature of a body by $1\ K$:

$$C_{th} = \rho V c_p \text{ (J/K)}$$

Volumetric heat capacity:

$$\rho c_p \text{ (J/m}^3\text{K)}$$

- **Conduction heat transfer:** Heat transfer through a material due to molecular/atomic interactions without bulk motion of matter.
- **Thermal resistance:** Resistance offered to heat flow:

$$R_{th} = \frac{\Delta T}{Q}$$

For plane wall:

$$R_{th} = \frac{L}{kA} \text{ (K/W)}$$

- **Q.** Define thermal capacity and thermal diffusivity of a material. **(WINTER 2025, 3 Marks)**

Answer:

Thermal diffusivity:

$$\boxed{\alpha = \frac{k}{\rho c_p}}$$

Units: $\boxed{m^2/s}$

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For plane wall:

$$R_{th} = \frac{L}{kA} \text{ (K/W)}$$

- **Q.** Define: 1) Conduction heat transfer 2) Thermal diffusivity 3) Thermal resistance. **(SUMMER 2026, 3 Marks)**

Answer:

Thermal diffusivity:

$$\boxed{\alpha = \frac{k}{\rho c_p}}$$

Units: $\boxed{m^2/s}$

- **Significance:** It indicates how quickly a temperature disturbance diffuses through a material relative to its ability to store heat. Higher α means faster temperature response and quicker propagation of heat.
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For plane wall:

$$R_{th} = \frac{L}{kA} \text{ (K/W)}$$

Other Important Questions:

1. Fourier's Law & Conduction Basics:

- **Q.** State Fourier's law and give units. **(SUMMER 2022)**

Answer:

Fourier's law:

$$\boxed{q_x = -kA\frac{dT}{dx}}$$

Units: q_x in W; k in W/m·K; $\frac{dT}{dx}$ in K/m.

- **Q.** Arrange materials (Copper, Water, Air, Wood) in descending order of thermal conductivity. **(SUMMER 2024)**

Answer:

Descending order of thermal conductivity:

$$\boxed{\text{Copper} > \text{Water} > \text{Wood} > \text{Air}}$$

- **Q.** How does the heat conduction differ from convection? **(WINTER 2025, 3 Marks)**

Answer:

Conduction vs convection:

Basis	Conduction	Convection
Medium	Solid/stationary fluid	Moving fluid
Mechanism	Molecular/atomic diffusion	Bulk fluid motion + conduction
Law	Fourier's law	Newton's law of cooling
Rate depends on	k, area, temperature gradient	h, area, temperature difference

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2. Plane Wall & Composite Wall – Derivations:

- **Q.** Derive expression for temperature distribution under one dimensional steady state heat conduction for plane wall. **(WINTER 2025, 7 Marks)**

Answer:

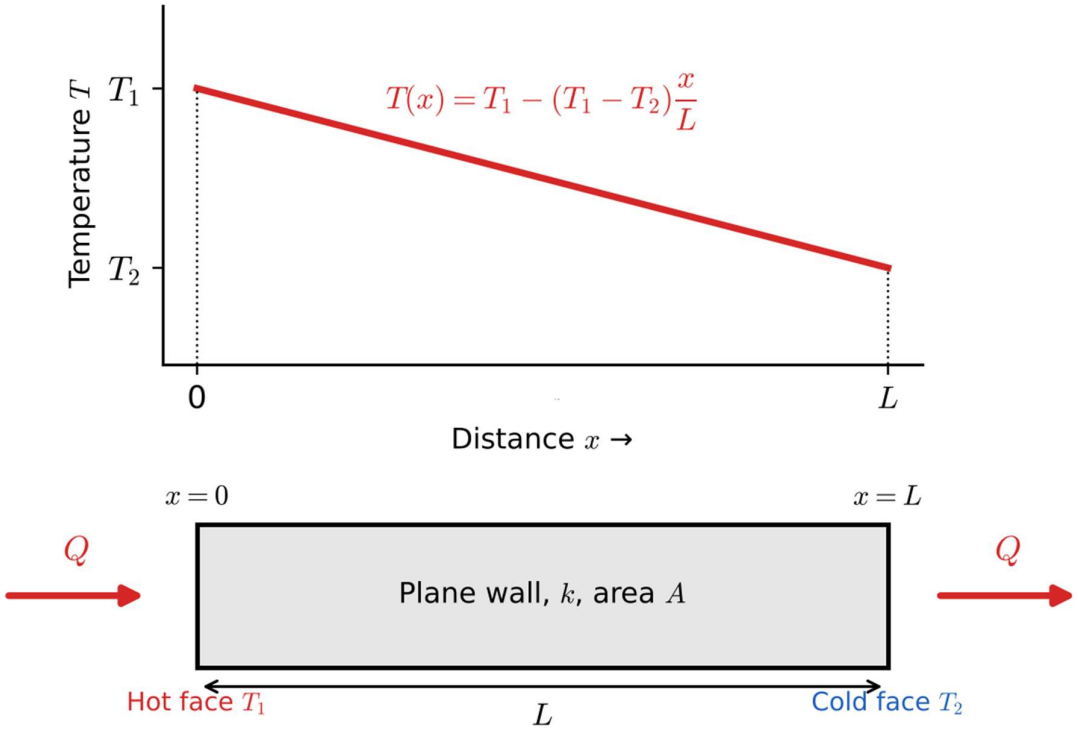


Fig. 4: Plane wall of thickness L with linear temperature distribution between T_1 and T_2

Steps:

1. For 1D steady conduction with no heat generation:

$$\frac{d^2T}{dx^2} = 0$$

2. Integrate twice:

$$T(x) = C_1x + C_2$$

3. Boundary conditions:

$$T(0) = T_1, \quad T(L) = T_2$$

4. Apply:

$$C_2 = T_1, \quad C_1 = \frac{T_2 - T_1}{L}$$

5. Temperature distribution:

$$\boxed{T(x) = T_1 - \frac{T_1 - T_2}{L}x}$$

or

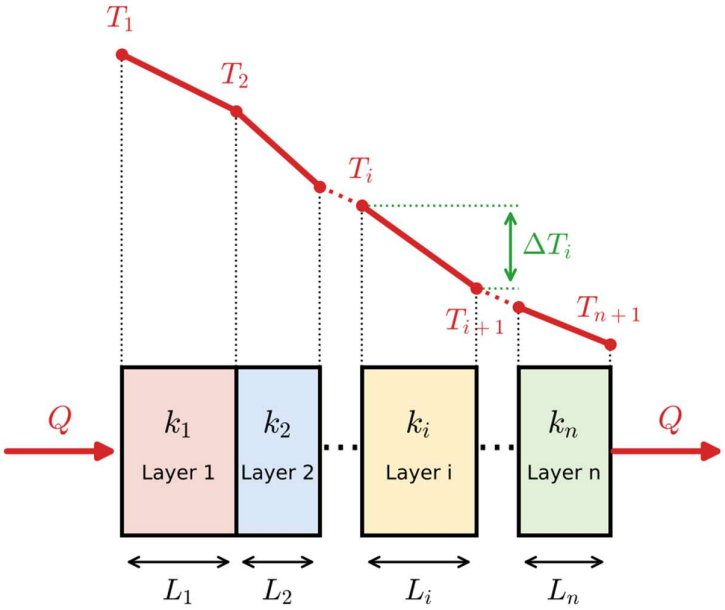
$$\boxed{\frac{T(x) - T_1}{T_2 - T_1} = \frac{x}{L}}$$

Heat transfer rate:

$$\boxed{Q = kA\frac{T_1 - T_2}{L}}$$

○ **Q.** Derive the heat conduction equation through composite wall. **(WINTER 2025, 7 Marks OR)**

Answer:



$$Q = \frac{T_1 - T_{n+1}}{\Sigma\, L_i/(k_i A)}, \quad \Delta T_i = \frac{Q L_i}{k_i A}$$

Fig. 5: Composite wall of n layers in series with temperature drop across each layer

Steps:

1. Write resistance of each layer:

$$R_i = \frac{L_i}{k_i A}$$

2. For series layers, total resistance:

$$R_{\text{total}} = \sum_{i=1}^{n} \frac{L_i}{k_i A}$$

3. Heat flow:

$$\boxed{Q = \frac{T_1 - T_{n+1}}{\sum_{i=1}^{n} \frac{L_i}{k_i A}}}$$

4. Intermediate interface temperatures:

$$T_2 = T_1 - Q\frac{L_1}{k_1 A}$$

$$T_3 = T_2 - Q\frac{L_2}{k_2 A}$$

and so on.

Overall heat transfer coefficient:

$$\boxed{U = \frac{1}{\sum_{i=1}^{n} \frac{L_i}{k_i}}}$$

for unit area.

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3. Insulation & Composite Wall Numericals:

- **Q.** A steel pipe of 20 mm inner diameter and 2 mm thickness is covered with 20 mm thick of fiber glass insulation (k = 0.05 W/m-deg). If the inside and outside convective coefficients are 10 W/m²-deg and 5 W/m²-deg, calculate the overall heat transfer coefficient based on inside diameter of pipe. **(SUMMER 2026)**

Answer:

Given:

- Inner diameter of steel pipe, $d_i = 20\,mm$
- Inner radius, $r_i = 0.010\,m$
- Pipe thickness $= 2\,mm$
- Pipe outer radius, $r_2 = 0.012\,m$
- Insulation thickness $= 20\,mm$
- Insulation outer radius, $r_o = 0.032\,m$
- Insulation conductivity, $k_{ins} = 0.05\,W/m \cdot K$
- Inside convective coefficient, $h_i = 10\,W/m^2K$
- Outside convective coefficient, $h_o = 5\,W/m^2K$

To find: Overall heat transfer coefficient based on inside diameter, U_i.

Formula:

$$U_i = \frac{1}{\frac{1}{h_i} + \frac{r_i}{k_{pipe}}\ln\frac{r_2}{r_i} + \frac{r_i}{k_{ins}}\ln\frac{r_o}{r_2} + \frac{r_i}{r_o}\frac{1}{h_o}}$$

Since k_{pipe} is not given, pipe wall resistance is negligible compared with insulation and convection.

Solution:

$$\frac{1}{h_i} = \frac{1}{10} = 0.1$$

$$\frac{r_i}{k_{ins}}\ln\frac{r_o}{r_2} = \frac{0.010}{0.05}\ln\frac{0.032}{0.012} = 0.2\ln(2.6667)$$

$$= 0.2(0.9808) = 0.1962$$

$$\frac{r_i}{r_o}\frac{1}{h_o} = \frac{0.010}{0.032} \times \frac{1}{5} = 0.3125 \times 0.2 = 0.0625$$

$$\sum R = 0.1 + 0.1962 + 0.0625 = 0.3587$$

$$U_i = \frac{1}{0.3587} = 2.79\ W/m^2K$$

Final Answer:

$$\boxed{U_i \approx 2.79\ W/m^2K \approx 2.8\ W/m^2K}$$

- **Q.** Calculate the heat flow rate through the composite wall as shown in figure. Assume 1D flow and take ka = 150 W/m-deg, kb = 30 W/m-deg, kc = 65 W/m-deg, kd = 50 W/m-deg. [Diagram required – refer to the original question paper of SUMMER 2026] **(SUMMER 2026 OR)**

Answer:

Given:

$$k_a = 150,\ k_b = 30,\ k_c = 65,\ k_d = 50\ \text{W/m} \cdot \text{K}$$

- **Note:** The figure is required to know which layers are in series/parallel and their lengths/areas. Use electrical analogy.

If layers are in series:

$$Q = \frac{\Delta T}{\frac{L_a}{k_a A_a} + \frac{L_b}{k_b A_b} + \frac{L_c}{k_c A_c} + \frac{L_d}{k_d A_d}}$$

If two branches are in parallel:

$$R_{eq} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$Q = \frac{\Delta T}{R_{\text{total}}}$$

- **Final Answer:** Substitute L and A from the given figure into the above resistance network to obtain Q.
