Chapter 2: Fins (Extended Surfaces)

Repeated Questions:

1. Fin Temperature Distribution & Heat Transfer Rate:

- **Q.** Derive the temperature distribution and heat transfer rate for a fin (insulated tip/losing heat). **(WINTER 2022, 3 Marks - Efficiency & Effectiveness; SUMMER 2025, 7 Marks)**

Answer:

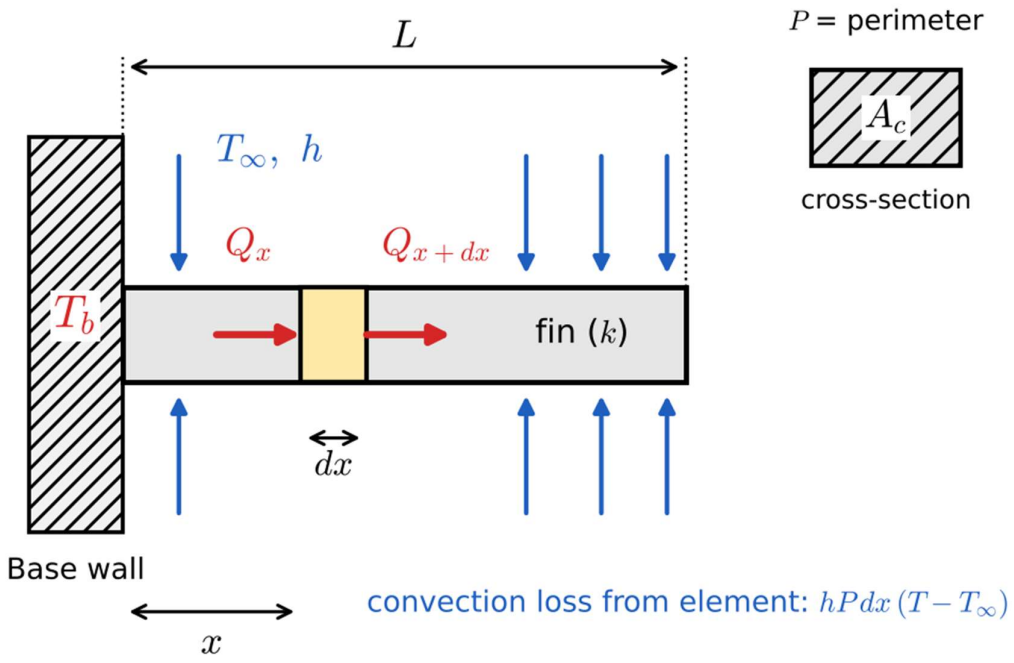


Fig. 6: Fin of length L with differential element dx (energy balance)

- **Assumptions:** 1D steady conduction along fin; constant k, h, A_c, P; uniform base temperature; negligible radiation; no heat generation.

Steps:

1. Apply energy balance on differential element dx:

$$Q_x = Q_{x+dx} + hP\,dx\,(T - T_\infty)$$

2. Using Fourier's law:

$$Q_x = -kA_c\frac{dT}{dx}$$

$$Q_{x+dx} = Q_x + \frac{dQ_x}{dx}dx$$

3. Substitute and simplify:

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = 0$$

Let excess temperature $\theta = T - T_\infty$, and fin parameter:

$$m = \sqrt{\frac{hP}{kA_c}}$$

$$\boxed{\frac{d^2\theta}{dx^2} - m^2\theta = 0}$$

General solution:

$$\theta = C_1 e^{mx} + C_2 e^{-mx}$$

or

$$\theta = C_1 \cosh mx + C_2 \sinh mx$$

Case A: Insulated tip

Boundary conditions:

$$\text{At } x = 0,\ \theta = \theta_b = T_b - T_\infty$$

$$\text{At } x = L,\ \frac{d\theta}{dx} = 0$$

Applying:

$$\boxed{\frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL}}$$

$$\boxed{\frac{T - T_\infty}{T_b - T_\infty} = \frac{\cosh m(L-x)}{\cosh mL}}$$

Heat transfer rate:

$$\boxed{Q_{fin} = \sqrt{hPkA_c}\ \theta_b \tanh mL}$$

Case B: Convection from fin tip

Boundary condition at tip:

$$-k\frac{d\theta}{dx}|_{x=L} = h\theta(L)$$

Result:

$$\boxed{\frac{\theta}{\theta_b} = \frac{\cosh m(L-x) + \frac{h}{mk}\sinh m(L-x)}{\cosh mL + \frac{h}{mk}\sinh mL}}$$

$$\boxed{Q_{fin} = \sqrt{hPkA_c}\ \theta_b \frac{\sinh mL + \frac{h}{mk}\cosh mL}{\cosh mL + \frac{h}{mk}\sinh mL}}$$

For insulated tip, put $h/mk = 0$, giving the earlier result.

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2. Fin Efficiency & Effectiveness:

- **Q.** Define/Explain Fin Efficiency and Fin Effectiveness. **(WINTER 2022, 3 Marks; WINTER 2025, 7 Marks; SUMMER 2026, 3 Marks OR)**

Answer:

Fin efficiency:

$$\boxed{\eta_{fin} = \frac{Q_{fin}}{Q_{max}} = \frac{Q_{fin}}{hA_{fin}\theta_b}}$$

where A_{fin} is total fin surface area.

For insulated tip:

$$\boxed{\eta_{fin} = \frac{\tanh mL}{mL}}$$

For convection tip:

$$\eta_{fin} = \frac{1}{mL}\frac{\sinh mL + \frac{h}{mk}\cosh mL}{\cosh mL + \frac{h}{mk}\sinh mL}$$

Fin effectiveness:

$$\varepsilon_{fin} = \frac{Q_{fin}}{hA_b\theta_b}$$

where A_b is base area without fin.

For insulated tip:

$$\varepsilon_{fin} = \sqrt{\frac{kP}{hA_c}}\tanh mL = \frac{mk}{h}\tanh mL$$

Exam points:

Parameter	Meaning	Desired value
Fin efficiency	Actual fin heat transfer / ideal heat transfer if whole fin at base temperature	As high as possible, usually < 1
Fin effectiveness	Fin heat transfer / heat transfer from bare base area	Should be > 2 for practical use

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- **Q.** Explain following: 1. Fin efficiency 2. Fin effectiveness. **(WINTER 2025, 7 Marks)**

Answer:

Fin efficiency:

$$\eta_{fin} = \frac{Q_{fin}}{Q_{ma}} = \frac{Q_{fin}}{hA_{fin}\theta_b}$$

where A_{fin} is total fin surface area.

For insulated tip:

$$\eta_{fin} = \frac{\tanh mL}{mL}$$

For convection tip:

$$\eta_{fin} = \frac{1}{mL}\frac{\sinh mL + \frac{h}{mk}\cosh mL}{\cosh mL + \frac{h}{mk}\sinh mL}$$

Fin effectiveness:

$$\varepsilon_{fin} = \frac{Q_{fin}}{hA_b\theta_b}$$

where A_b is base area without fin.

For insulated tip:

$$\varepsilon_{fin} = \sqrt{\frac{kP}{hA_c}}\tanh mL = \frac{mk}{h}\tanh mL$$

Exam points:

Parameter	Meaning	Desired value
Fin efficiency	Actual fin heat transfer / ideal heat transfer if whole fin at base temperature	As high as possible, usually < 1
Fin effectiveness	Fin heat transfer / heat transfer from bare base area	Should be > 2 for practical use

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- **Q.** Define Fin Efficiency, Fin effectiveness with expressions. **(SUMMER 2026, 3 Marks OR)**

Answer:

Fin efficiency:

$$\boxed{\eta_{fin} = \frac{Q_{fin}}{Q_{max}} = \frac{Q_{fin}}{hA_{fin}\theta_b}}$$

where A_{fin} is total fin surface area.

For insulated tip:

$$\boxed{\eta_{fin} = \frac{\tanh mL}{mL}}$$

For convection tip:

$$\boxed{\eta_{fin} = \frac{1}{mL}\frac{\sinh mL + \frac{h}{mk}\cosh mL}{\cosh mL + \frac{h}{mk}\sinh mL}}$$

Fin effectiveness:

$$\boxed{\varepsilon_{fin} = \frac{Q_{fin}}{hA_b\theta_b}}$$

where A_b is base area without fin.

For insulated tip:

$$\boxed{\varepsilon_{fin} = \sqrt{\frac{kP}{hA_c}}\tanh mL = \frac{mk}{h}\tanh mL}$$

Exam points:

Parameter	Meaning	Desired value
Fin efficiency	Actual fin heat transfer / ideal heat transfer if whole fin at base temperature	As high as possible, usually < 1
Fin effectiveness	Fin heat transfer / heat transfer from bare base area	Should be > 2 for practical use

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3. Fin Numericals:

- **Q.** Numericals on Fin Heat Dissipation and Efficiency. **(SUMMER 2022, 7 Marks; WINTER 2022, 7 Marks; WINTER 2023, 7 Marks; SUMMER 2024, 7 Marks; WINTER 2024, 7 Marks OR; SUMMER 2026, 7 Marks OR)**

Answer:

Given:

- Thermal conductivity, $k = 30\ W/m \cdot K$
- Diameter, $d = 1\ cm = 0.01\ m$
- Length, $L = 5\ cm = 0.05\ m$
- Base temperature, $T_b = 100°C$
- Fluid temperature, $T_\infty = 30°C$
- Convection coefficient, $h = 50\ W/m^2K$
- Tip is insulated.

To find: Fin efficiency η_{fin}, tip temperature T_L, heat dissipation Q_{fin}.

Formula:

$$P = \pi d, \quad A_c = \frac{\pi d^2}{4}$$

$$m = \sqrt{\frac{hP}{kA_c}}, \quad mL$$

$$\eta_{fin} = \frac{\tanh mL}{mL}$$

$$\frac{T_L - T_\infty}{T_b - T_\infty} = \frac{1}{\cosh mL}$$

$$Q_{fin} = \sqrt{hPkA_c}\,(T_b - T_\infty)\tanh mL$$

Solution:

Perimeter:

$$P = \pi(0.01) = 0.031416\ m$$

Cross-sectional area:

$$A_c = \frac{\pi(0.01)^2}{4} = 7.854 \times 10^{-5}\ m^2$$

Fin parameter:

$$m = \sqrt{\frac{50(0.031416)}{30(7.854 \times 10^{-5})}} = \sqrt{666.67} = 25.82\ m^{-1}$$

$$mL = 25.82(0.05) = 1.291$$

Fin efficiency:

$$\tanh(1.291) = 0.8594$$

$$\eta_{fin} = \frac{0.8594}{1.291} = 0.6657$$

$$\boxed{\eta_{fin} \approx 66.6\%}$$

Tip temperature:

$$\cosh(1.291) = 1.956$$

$$\frac{T_L - 30}{100 - 30} = \frac{1}{1.956} = 0.5113$$

$$T_L = 30 + 70(0.5113) = 65.79°C$$

$$\boxed{T_L \approx 65.8°C}$$

Heat dissipation:

$$\sqrt{hPkA_c} = \sqrt{50(0.031416)(30)(7.854 \times 10^{-5})} = 0.06084\ W/K$$

$$Q_{fin} = 0.06084(70)(0.8594) = 3.66\ W$$

$$\boxed{Q_{fin} \approx 3.66\ W}$$

Final Answer:

$$\boxed{\eta_{fin} \approx 66.6\%, \quad T_L \approx 65.8°C, \quad Q_{fin} \approx 3.66\ W}$$

- **Q.** A steel rod (k = 30 W/m-deg), 1 cm in diameter and 5 cm long protrudes from a wall which is maintained at 100 °C. The rod is insulated at its tip and is exposed to an environment with h = 50 W/m²-deg and ta = 30 °C. Calculate the fin efficiency, temperature at the tip of fin and the rate of heat dissipation. **(SUMMER 2026, 7 Marks OR)**

Answer:

Given:

- Thermal conductivity, $k = 30\ W/m \cdot K$
- Diameter, $d = 1\ cm = 0.01\ m$
- Length, $L = 5\ cm = 0.05\ m$
- Base temperature, $T_b = 100°C$
- Fluid temperature, $T_\infty = 30°C$
- Convection coefficient, $h = 50\ W/m^2K$
- Tip is insulated.

To find: Fin efficiency η_{fin}, tip temperature T_L, heat dissipation Q_{fin}.

Formula:

$$P = \pi d, \quad A_c = \frac{\pi d^2}{4}$$

$$m = \sqrt{\frac{hP}{kA_c}}, \quad mL$$

$$\eta_{fin} = \frac{\tanh mL}{mL}$$

$$\frac{T_L - T_\infty}{T_b - T_\infty} = \frac{1}{\cosh mL}$$

$$Q_{fin} = \sqrt{hPkA_c}\,(T_b - T_\infty)\tanh mL$$

Solution:

Perimeter:

$$P = \pi(0.01) = 0.031416\ m$$

Cross-sectional area:

$$A_c = \frac{\pi(0.01)^2}{4} = 7.854 \times 10^{-5}\ m^2$$

Fin parameter:

$$m = \sqrt{\frac{50(0.031416)}{30(7.854 \times 10^{-5})}} = \sqrt{666.67} = 25.82\ m^{-1}$$

$$mL = 25.82(0.05) = 1.291$$

Fin efficiency:

$$\tanh(1.291) = 0.8594$$
$$\eta_{fin} = \frac{0.8594}{1.291} = 0.6657$$
$$\boxed{\eta_{fin} \approx 66.6\%}$$

Tip temperature:

$$\cosh(1.291) = 1.956$$
$$\frac{T_L - 30}{100 - 30} = \frac{1}{1.956} = 0.5113$$
$$T_L = 30 + 70(0.5113) = 65.79°C$$
$$\boxed{T_L \approx 65.8°C}$$

Heat dissipation:

$$\sqrt{hPkA_c} = \sqrt{50(0.031416)(30)(7.854 \times 10^{-5})} = 0.06084\ W/K$$
$$Q_{fin} = 0.06084(70)(0.8594) = 3.66\ W$$
$$\boxed{Q_{fin} \approx 3.66\ W}$$

Final Answer:

$$\boxed{\eta_{fin} \approx 66.6\%, \quad T_L \approx 65.8°C, \quad Q_{fin} \approx 3.66\ W}$$

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4. Justification & Selection of Fins:

- **Q.** Factors/Justifications for using fins. **(SUMMER 2022, 3 Marks - Factors for optimum design; WINTER 2024, 3 Marks - "Fins may decrease heat transfer"; WINTER 2024, 4 Marks - When are fins not useful?; WINTER 2025, 4 Marks - Criteria of selection of fins)**

Answer:

Fins are justified when:

- Convection coefficient h is low — fins compensate by increasing surface area.
- Thermal conductivity k is high — fin material conducts heat effectively to the extended surface.
- Perimeter-to-area ratio P/A_c is large — thin fins give more heat transfer per unit volume.
- Fin effectiveness $\varepsilon_{fin} > 2$ — otherwise fin is not practically worthwhile.
- Fin efficiency is reasonable — very long fins have low efficiency and wasted material.
- Space, weight, cost, and corrosion constraints are acceptable.

When fins are not useful / may decrease heat transfer:

- When h is very high, e.g., boiling or condensation surfaces.
- When fin material has very low k.
- When fin is too long/thin such that efficiency is very low.
- When fin effectiveness is less than 1 — added conduction resistance dominates.
- When pressure drop, fouling, weight, or cost penalties outweigh benefit.
- **Criteria for selection of fins:** high k, low h, large P/A_c, moderate length, good corrosion resistance, low cost, and $\varepsilon_{fin} > 2$.

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5. Thermometer Well:

- **Q.** Estimation of error in temperature measurement in a thermometer well. **(SUMMER 2022, 7 Marks; SUMMER 2025, 4 Marks)**

Answer:

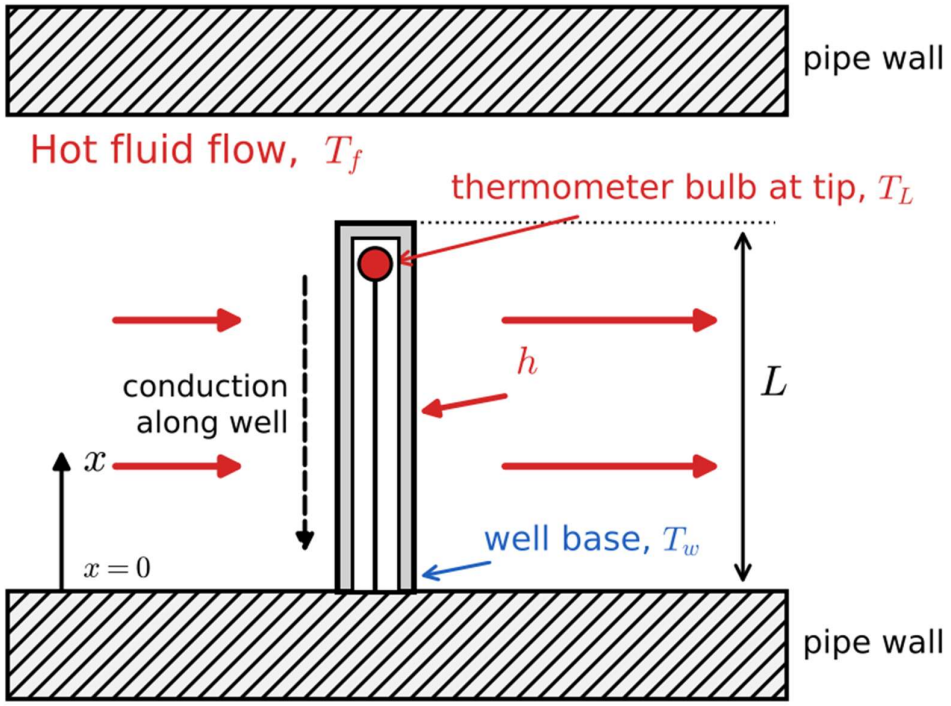


Fig. 7: Thermometer well projecting into a hot fluid stream (treated as a fin)

- **Concept:** The well acts as a fin. Heat conducts along the well from the wall to the fluid, so the thermometer reads a temperature different from the true fluid temperature. This causes measurement error.

Steps:

1. Treat well as a fin with base at wall temperature T_w and fluid at T_f.

Excess temperature:

$$\theta = T - T_f$$

Fin equation:

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0, \quad m = \sqrt{\frac{hP}{kA_c}}$$

For insulated tip:

$$\frac{\theta(x)}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL}$$

where $\theta_b = T_w - T_f$.

At thermometer bulb, $x = L$:

$$\boxed{\frac{T_L - T_f}{T_w - T_f} = \frac{1}{\cosh mL}}$$

Temperature measurement error:

$$\boxed{\Delta T_{error} = T_L - T_f = \frac{T_w - T_f}{\cosh mL}}$$

Magnitude:

$$\boxed{|\Delta T_{error}| = \frac{|T_w - T_f|}{\cosh mL}}$$

- **Conclusion:** Error decreases as mL increases, i.e., for longer well, higher h, smaller k, and larger P/A_c. If $mL > 3$, error becomes negligible.

Other Important Questions:

1. Fin Advantages & Applications:

- **Q.** Advantages and limitations of using fins. **(SUMMER 2025, 4 Marks)**

Answer:

Advantages:

- Increases heat transfer surface area without increasing base area.
- Enhances heat dissipation from compact surfaces.
- Improves cooling of engines, electronics, heat exchangers, and radiators.
- Can be designed to give high effectiveness with low material use.
- Useful where h is low, such as air-side heat transfer.

Limitations:

- Adds weight, volume, and cost.
- Increases pressure drop in fluid flow.
- Fin efficiency is always less than 1.
- Poor design may reduce heat transfer if effectiveness < 1.
- Susceptible to fouling, corrosion, and vibration problems.

- **Q.** List the applications of Fin and explain how a fin enhances heat transfer at the surface. **(SUMMER 2026, 4 Marks)**

Answer:

Applications:

- Air-cooled engine cylinders and cylinder heads.
- Radiators and condensers in automobiles and refrigeration.
- Heat exchangers and economizers.
- Cooling of electronic components and CPUs.
- Electric motor and transformer casings.
- Steam power plant air-cooled condensers.

How a fin enhances heat transfer:

- It extends the surface area from A_b to A_{fin}, increasing convective heat transfer.
- It provides a conduction path from the base to the extended surface.
- It is most effective when h is low and k is high.
- Heat transfer increases if fin effectiveness is greater than 1; practically $\varepsilon_{fin} > 2$ is preferred.
