Chapter 3: Transient Heat Conduction

Repeated Questions:

1. Lumped Capacitance Method:

- **Q.** Lumped Capacitance Method & its Applicability (Biot Number). **(WINTER 2023, 3 Marks; WINTER 2024, 4 Marks; SUMMER 2025, 3 Marks; WINTER 2025, 4 Marks)**

Answer:

- **Definition:** The lumped capacitance method assumes that the temperature of a body remains spatially uniform at any instant during a transient process, i.e., internal conduction resistance is negligible compared with external convection resistance.

Applicability – Biot number criterion:

$$\boxed{Bi = \frac{hL_c}{k} < 0.1}$$

where $L_c = \frac{V}{A_s}$ is the characteristic length.

Governing equation:

$$\rho V c_p \frac{dT}{dt} = -hA_s(T - T_\infty)$$

Solution:

$$\boxed{\frac{T - T_\infty}{T_i - T_\infty} = e^{-t/\tau}}$$

where time constant $\tau = \frac{\rho V \ _p}{hA_s}$.

Key points:

- Valid for small bodies or high-conductivity materials.
- Internal temperature gradients are neglected.
- Used for quick cooling/heating estimates, e.g., thermocouples, small metal parts.

- **Q.** What is lumped heat capacity analysis? List out the assumptions made in lumped heat capacity analysis. **(WINTER 2025, 4 Marks)**

Answer:

- **Definition:** The lumped capacitance method assumes that the temperature of a body remains spatially uniform at any instant during a transient process, i.e., internal conduction resistance is negligible compared with external convection resistance.

Applicability – Biot number criterion:

$$\boxed{Bi = \frac{hL_c}{k} < 0.1}$$

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Governing equation:

$$\rho V c_p \frac{dT}{dt} = -hA_s(T - T_\infty)$$

Solution:

$$\boxed{\frac{T - T_\infty}{T_i - T_\infty} = e^{-t/\tau}}$$

where time constant $\tau = \frac{\rho V c_p}{hA_s}$.

Key points:

- Valid for small bodies or high-conductivity materials.
- Internal temperature gradients are neglected.
- Used for quick cooling/heating estimates, e.g., thermocouples, small metal parts.
- Uniform temperature throughout the body at any instant.
- Negligible internal conduction resistance (i.e., $Bi < 0.1$).
- Constant thermophysical properties (ρ, c_p, k).
- Convection heat transfer at the surface with constant h and T_∞.
- No internal heat generation.
- Radiation effects negligible (or included separately).

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2. Biot Number & Fourier Number:

- **Q.** Define and explain Biot Number and Fourier Number. **(WINTER 2024, 4 Marks; SUMMER 2026, 3 Marks)**

Answer:

Biot number:

$$\boxed{Bi = \frac{hL_c}{k}}$$

- **Physical significance:** Ratio of internal conduction resistance to external convection resistance.
- $Bi < 0.1$ → lumped capacitance valid.

Fourier number:

$$\boxed{Fo = \frac{\alpha t}{L_c^2}}$$

- **Physical significance:** Dimensionless time; ratio of heat conducted to heat stored.
- For lumped system: $\theta = \frac{T-T_\infty}{T_i-T_\infty} = e^{-Bi \cdot Fo}$ (with $L_c = V/A_s$).

- **Q.** Define Fourier number and Biot number in context of lumped system analysis. **(SUMMER 2026, 3 Marks)**

Answer:

Biot number:

$$\boxed{Bi = \frac{hL_c}{k}}$$

- **Physical significance:** Ratio of internal conduction resistance to external convection resistance.
- $Bi < 0.1$ → lumped capacitance valid.

Fourier number:

$$\boxed{Fo = \frac{\alpha t}{L_c^2}}$$

- **Physical significance:** Dimensionless time; ratio of heat conducted to heat stored.
- For lumped system: $\theta = \frac{T-T_\infty}{T_i-T_\infty} = e^{-B \cdot Fo}$ (with $L_c = V/A_s$).

- **Q.** Numericals based on lumped system analysis (Biot number and Fourier number). **(SUMMER 2023; WINTER 2024)**

Answer:

- **Problem:** A small copper sphere of diameter $5\,cm$, initially at $300°C$, is suddenly immersed in a fluid at $30°C$ with convection coefficient $h = 100\,W/m^2K$. Properties: $k = 380\,W/m \cdot K$, $\rho = 8800\,kg/m^3$, $c_p = 400\,J/kg \cdot K$. Find the time required for the sphere to reach $100°C$.

Given:

- Diameter $d = 0.05\,m$
- Initial temperature $T_i = 300°C$
- Fluid temperature $T_\infty = 30°C$
- Final temperature $T = 100°C$
- $h = 100\,W/m^2K$
- $k = 380\,W/m \cdot K$, $\rho = 8800\,kg/m^3$, $c_p = 400\,J/kg \cdot K$

To find: Time t

Formula:

$$Bi = \frac{hL_c}{k}, \quad L_c = \frac{V}{A_s} = \frac{d}{6} \text{ (for sphere)}$$

$$\frac{T - T_\infty}{T_i - T_\infty} = e^{-t/\tau}, \quad \tau = \frac{\rho c_p L_c}{h}$$

Solution:

Characteristic length:

$$L_c = \frac{d}{6} = \frac{0.05}{6} = 0.008333\,m$$

Biot number:

$$Bi = \frac{100 \times 0.008333}{380} = 0.00219 < 0.1$$

→ Lumped capacitance method is valid.

Time constant:

$$\tau = \frac{8800 \times 400 \times 0.008333}{100} = 293.3\,s$$

Temperature ratio:

$$\frac{T - T_\infty}{T_i - T_\infty} = \frac{100 - 30}{300 - 30} = \frac{70}{270} = 0.2593$$

Solve for t:

$$e^{-t/\tau} = 0.2593$$

$$-\frac{t}{\tau} = \ln(0.2593) = -1.350$$

$$t = 1.350 \times 293.3 = 396\,s$$

Final Answer:

$$\boxed{t \approx 396 \text{ s} \approx 6.6 \text{ min}}$$

Other Important Questions:

1. Newtonian Heating/Cooling & 1D Transient Conduction:

- **Q.** Derive the equation for temperature during Newtonian heating/cooling. **(SUMMER 2022, 7 Marks)**

Answer:

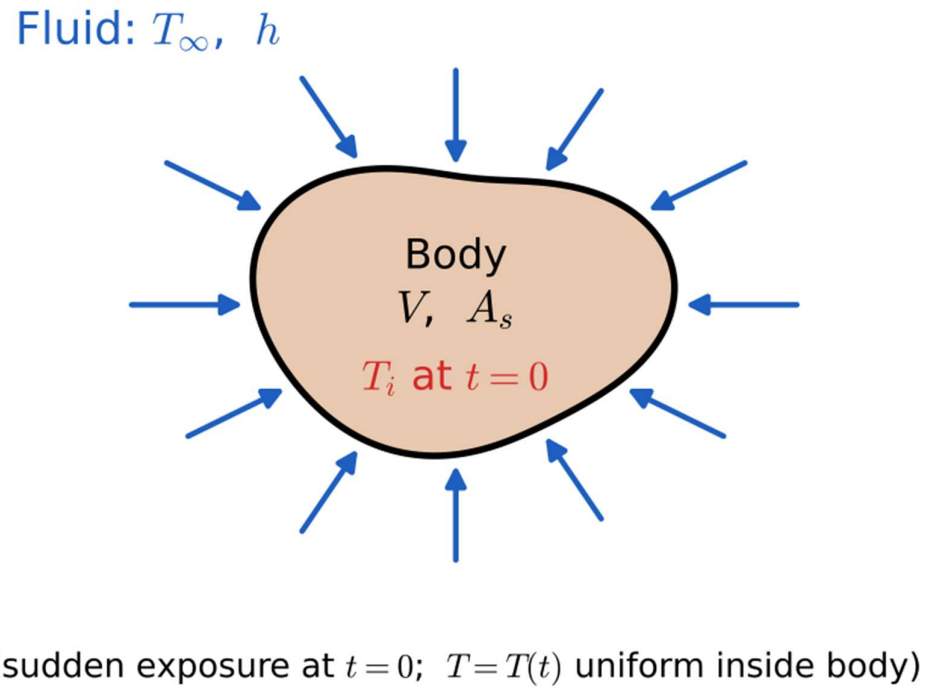


Fig. 8: Small body of volume V and surface area A_s suddenly exposed to fluid at T_∞ (lumped system)

- **Assumptions:** Lumped system ($Bi < 0.1$), constant properties, no heat generation.

Steps:

1. Energy balance on the body:

$$\text{Rate of heat loss} = \text{Rate of decrease of internal energy}$$

$$hA_s(T - T_\infty) = -\rho V c_p \frac{dT}{dt}$$

2. Rearrange:

$$\frac{dT}{T - T_\infty} = -\frac{hA_s}{\rho V c_p} dt$$

3. Let excess temperature $\theta = T - T_\infty$, $d\theta = dT$:

$$\frac{d\theta}{\theta} = -\frac{hA_s}{\rho V c_p} dt$$

4. Integrate from $t = 0$ ($\theta_i = T_i - T_\infty$) to t ($\theta = T - T_\infty$):

$$\ln \frac{\theta}{\theta_i} = -\frac{hA_s}{\rho V c_p} t$$

5. Exponentiate:

$$\boxed{\frac{T - T_\infty}{T_i - T_\infty} = e^{-t/\tau}}$$

where

$$\boxed{\tau = \frac{\rho V c_p}{h A_s} = \frac{\rho c_p L_c}{h}}$$

is the time constant.

Final result:

$$\boxed{T(t) = T_\infty + (T_i - T_\infty)e^{-t/\tau}}$$

- **Q.** Derive the governing equation for 1D transient heat conduction in a plane wall. **(SUMMER 2025, 3 Marks OR)**

Answer:

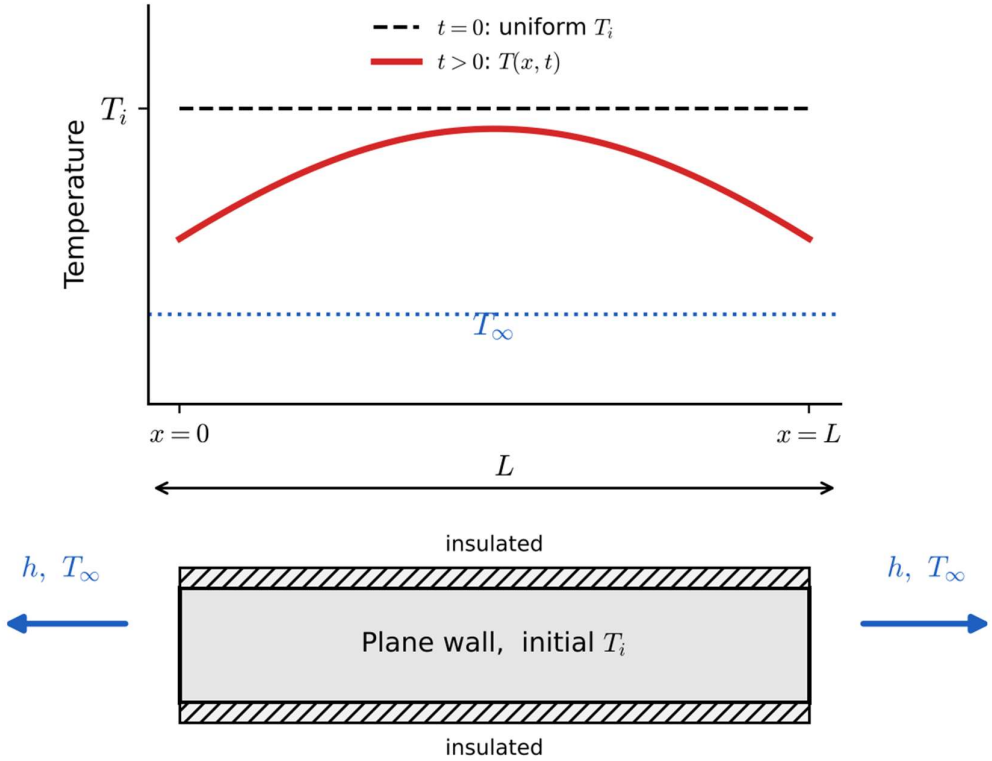


Fig. 9: Plane wall of thickness L, insulated on the sides, with convection on both surfaces (transient conduction)

- **Assumptions:** 1D conduction in x-direction, constant properties, no heat generation.

Steps:

1. Start from general heat conduction equation in Cartesian coordinates:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}_g}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

2. For 1D (y, z variations zero) and no heat generation ($\dot{q}_g = 0$):

$$\boxed{\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}}$$

Initial condition:

$$T(x, 0) = T_i$$

Boundary conditions (convection at both surfaces):

$$-k\frac{\partial T}{\partial x}|_{x=0} = h[T_\infty - T(0, t)]$$

$$-k\frac{\partial T}{\partial x}|_{x=L} = h[T(L, t) - T_\infty]$$

- **Note:** Solution requires separation of variables or use of Heisler charts.
