Chapter 4: Convection & Boundary Layer Theory

Repeated Questions:

1. Dimensionless Numbers:

- **Q.** Define and explain Physical Significance of Dimensionless Numbers (Nu, Re, Pr, Gr, Bi, Pe). **(WINTER 2022, 4 Marks; SUMMER 2023, 4 Marks OR; SUMMER 2025, 7 Marks; WINTER 2025, 4 Marks OR)**

Answer:

Reynolds number:

$$Re = \frac{\rho V L}{\mu} = \frac{V L}{\nu}$$

- **Significance:** Ratio of inertia force to viscous force; decides laminar/turbulent flow.

Prandtl number:

$$Pr = \frac{\mu c_p}{k} = \frac{\nu}{\alpha}$$

- **Significance:** Ratio of momentum diffusivity to thermal diffusivity; controls relative thickness of velocity and thermal boundary layers.

Nusselt number:

$$Nu = \frac{hL}{k}$$

- **Significance:** Ratio of convective heat transfer to conduction heat transfer across same fluid layer; dimensionless heat transfer coefficient.

Grashof number:

$$Gr = \frac{g\beta \Delta T L^3}{\nu^2}$$

- **Significance:** Ratio of buoyancy force to viscous force; governs free convection.

Biot number:

$$Bi = \frac{hL_c}{k_s}$$

- **Significance:** Ratio of internal conduction resistance to external convection resistance; $Bi < 0.1$ validates lumped analysis.

Peclet number:

$$Pe = Re \cdot Pr = \frac{VL}{\alpha}$$

- **Significance:** Ratio of advective heat transport to conductive heat transport.

- **Q.** Explain physical significance of Grashof number and Reynolds number. **(WINTER 2025, 4 Marks OR)**

Answer:

- **Grashof number Gr:** Represents buoyancy-driven free convection. Large Gr means buoyancy dominates viscosity and flow tends to turbulent free convection.
- **Reynolds number Re:** Represents forced-flow inertia versus viscous resistance. Small Re: laminar; large Re: turbulent.
- In free convection, Gr replaces Re as the flow-regime parameter; $GrPr$ is used instead of $RePr$.

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2. Free vs Forced Convection:

- **Q.** Differentiate between Free/Natural and Forced Convection. **(WINTER 2022, 4 Marks; SUMMER 2023, 3 Marks; SUMMER 2024, 3 Marks; WINTER 2025, 3 Marks)**

Answer:

Basis	Free/Natural convection	Forced convection
Driving force	Buoyancy due to density difference	External pump/fan/blower
Governing parameter	Gr, Pr	Re, Pr
Heat transfer coefficient	Lower	Higher

- **Q.** State comparison between natural and forced convection heat transfer. **(WINTER 2025, 3 Marks)**

Answer:

Basis	Free/Natural convection	Forced convection
Driving force	Buoyancy due to density difference	External pump/fan/blower
Governing parameter	Gr, Pr	Re, Pr
Heat transfer coefficient	Lower	Higher

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3. Dimensional Analysis (Buckingham π-theorem):

- **Q.** Using Dimensional Analysis (Buckingham π-theorem), derive the correlation for Forced Convection: Nu = f(Re, Pr). **(SUMMER 2024, 7 Marks; WINTER 2025, 7 Marks OR)**

Answer:

Variables:

$$h = f\left(V, L, \rho, \mu, c_p, k\right)$$

Total variables $n = 7$. Fundamental dimensions M, L, T, Θ: $m = 4$.

$$\text{Number of } \pi\text{-terms} = n - m = 7 - 4 = 3$$

Steps:

1. Write dimensions:

$$[h] = MT^{-3}\Theta^{-1}, \quad [V] = LT^{-1}, \quad [L] = L$$
$$[\rho] = ML^{-3}, \quad [\mu] = ML^{-1}T^{-1}, \quad \left[c_p\right] = L^2T^{-2}\Theta^{-1}, \quad [k] = MLT^{-3}\Theta^{-1}$$

2. Choose repeating variables: L, V, ρ, μ.
3. Form π_1 using h:

$$\pi_1 = hL^aV^b\rho^c\mu^d$$

Solving dimensions gives:

$$\boxed{\pi_1 = \frac{hL}{k} = Nu}$$

4. Form π_2 using k or directly from repeating variables:

$$\boxed{\pi_2 = \frac{\rho VL}{\mu} = Re}$$

5. Form π_3 using c_p:

$$\boxed{\pi_3 = \frac{\mu c_p}{k} = Pr}$$

Therefore:

$$\pi_1 = \phi(\pi_2, \pi_3)$$
$$\boxed{Nu = \phi(Re, Pr)}$$

Common power-law form:

$$\boxed{Nu = C\, Re^m Pr^n}$$

- **Q.** Using Dimensional Analysis (Buckingham π-theorem), derive the correlation for Free Convection: Nu = f(Gr, Pr). **(WINTER 2022, 7 Marks OR; SUMMER 2023, 7 Marks OR; SUMMER 2026, 7 Marks)**

Answer:

Variables:

$$h = f\left(L, \rho, \mu, c_p, k, g\beta\Delta T\right)$$

Total $n = 7$, $m = 4$, so $\pi = 3$.

Steps:

1. Repeating variables: L, ρ, μ, k.
2. First π-term:

$$\boxed{\pi_1 = \frac{hL}{k} = Nu}$$

3. Second π-term using buoyancy term $g\beta\Delta T$:

$$\boxed{\pi_2 = \frac{g\beta\Delta T L^3 \rho^2}{\mu^2} = \frac{g\beta\Delta T L^3}{\nu^2} = Gr}$$

4. Third π-term using c_p:

$$\boxed{\pi_3 = \frac{\mu c_p}{k} = Pr}$$

Hence:

$$\boxed{Nu = \phi(Gr, Pr)}$$

Common form:

$$\boxed{Nu = C(GrPr)^n}$$

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4. Convection Numericals:

- **Q.** Numericals on Free/Forced Convection using given correlations. **(SUMMER 2022, 7 Marks; SUMMER 2023, 7 Marks OR; WINTER 2023, 7 Marks)**

Answer:

When numerical data are given, use:

1. Evaluate fluid properties at film temperature:

$$T_f = \frac{T_s + T_\infty}{2}$$

2. Compute relevant dimensionless numbers:

$$Re = \frac{VL}{\nu}, \quad Pr = \frac{\mu c_p}{k}$$

or for free convection:

$$Gr = \frac{g\beta\Delta T L^3}{\nu^2}, \quad Ra = GrPr$$

3. Select correlation:

$$Nu = C\,Re^m Pr^n$$

or

$$Nu = C(GrPr)^n$$

4. Find heat transfer coefficient:

$$\boxed{h = \frac{Nu \cdot k}{L}}$$

5. Heat transfer rate:

$$\boxed{Q = hA(T_s - T_\infty)}$$

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5. Velocity & Thermal Boundary Layer:

- **Q.** Explain Velocity and Thermal Boundary Layer. **(SUMMER 2023, 7 Marks OR; SUMMER 2026, 3 Marks)**

Answer:

- **Hydrodynamic/velocity boundary layer:** The thin region near a solid surface where fluid velocity changes from zero at the wall (no-slip) to $0.99U_\infty$ at the outer edge. Thickness is δ.
- **Thermal boundary layer:** The region near the surface where temperature changes from wall temperature T_s to $0.99(T_\infty - T_s)$ at the outer edge. Thickness is δ_t.

Key points:

- Velocity boundary layer is due to viscosity and no-slip condition.
- Thermal boundary layer is due to heat transfer between wall and fluid.
- For $Pr \approx 1$, $\delta \approx \delta_t$.
- For $Pr > 1$, $\delta_t < \delta$; for $Pr < 1$, $\delta_t > \delta$.

Local convection coefficient:

$$h_x = \frac{-k_f\ \partial T/\ \partial y|_{y=0}}{T_s - T_\infty}$$

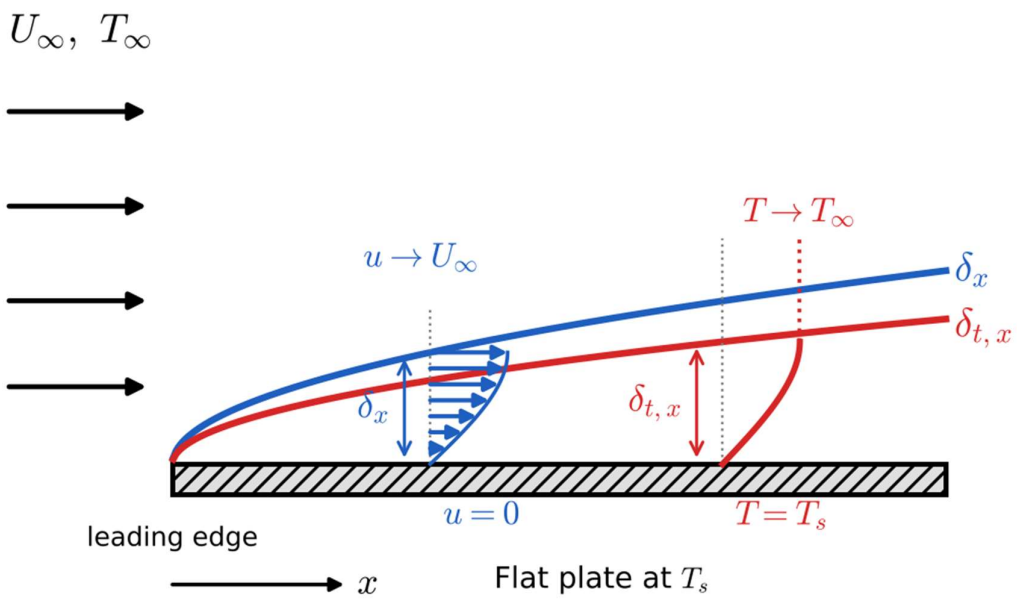


Fig. 10: Velocity and thermal boundary layers on a flat plate

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○ **Q.** Define thermal boundary layer and hydrodynamic boundary layer. **(SUMMER 2026, 3 Marks)**

Answer:

- **Hydrodynamic/velocity boundary layer:** The thin region near a solid surface where fluid velocity changes from zero at the wall (no-slip) to $0.99U_\infty$ at the outer edge. Thickness is δ.
- **Thermal boundary layer:** The region near the surface where temperature changes from wall temperature T_s to $0.99(T_\infty - T_s)$ at the outer edge. Thickness is δ_t.

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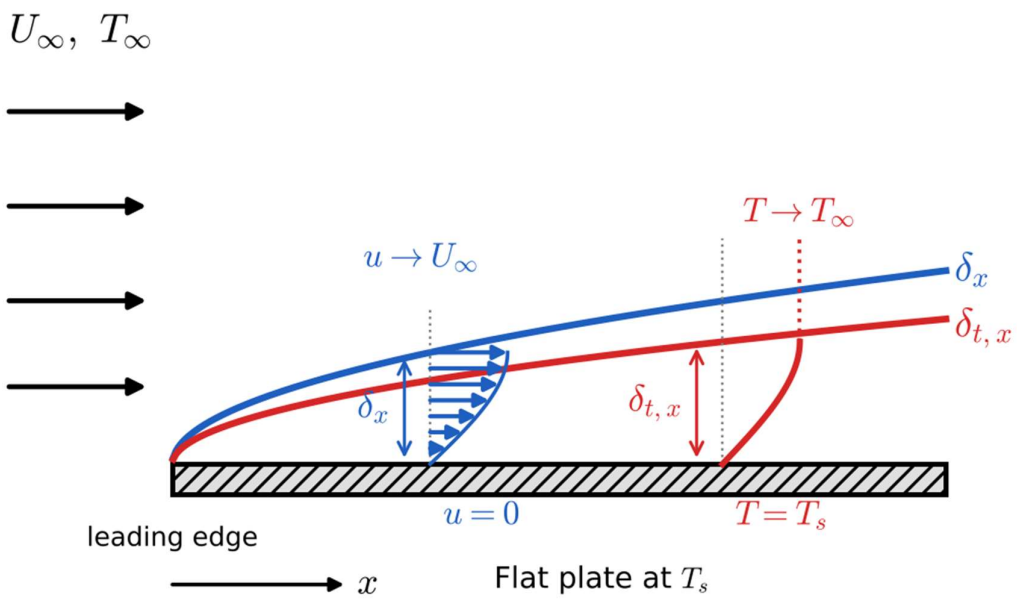


Fig. 11: Velocity and thermal boundary layers on a flat plate

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6. Von-Karman Integral Momentum Equation / Blasius Solution:

- **Q.** Explain Von-Karman integral momentum equation/Blasius solution. **(WINTER 2023, 7 Marks OR; SUMMER 2025, 4 Marks)**

Answer:

Von-Karman integral momentum equation:

$$\boxed{\frac{d}{dx}\int_0^\delta u\,(U-u)\,dy + \frac{dU}{dx}\int_0^\delta (U-u)\,dy = \frac{\tau_w}{\rho}}$$

For flat plate with constant free-stream velocity U:

$$\boxed{\frac{\tau_w}{\rho} = \frac{d}{dx}\int_0^\delta u\,(U-u)\,dy}$$

with

$$\tau_w = \mu \frac{\partial u}{\partial y}|_{y=0}$$

Blasius solution:

Similarity variable:

$$\eta = y\sqrt{\frac{U}{\nu x}}$$

Stream function:

$$\psi = \sqrt{\nu U x}\, f(\eta)$$

Velocity:

$$\frac{u}{U} = f'(\eta)$$

Blasius equation:

$$\boxed{2f''' + ff'' = 0}$$

Boundary conditions:

$$f(0) = 0, \quad f'(0) = 0, \quad f'(\infty) = 1$$

Results:

$$\boxed{\delta = \frac{5x}{\sqrt{Re_x}}}$$

$$\boxed{C_f = \frac{0.664}{\sqrt{Re_x}}}$$

$$\boxed{Nu_x = 0.332\, Re_x^{1/2} Pr^{1/3}}$$

Other Important Questions:

1. Convection Basics:

- **Q.** Explain Newton's law of cooling and convection heat transfer coefficient. **(WINTER 2025, 4 Marks)**

Answer:

$$\boxed{Q = hA(T_s - T_\infty)}$$

where h is the convection heat transfer coefficient in W/m^2K.

- h depends on fluid properties, flow velocity, geometry, and surface condition.
- Higher h means stronger convection.
- It is not a material property; it is evaluated from correlations using Nu.

- **Q.** Explain that heat transfer coefficient for free convection is lower than forced convection. **(WINTER 2025, 3 Marks OR)**

Answer:

- Forced convection has external flow velocity, producing a thinner boundary layer and steeper temperature gradient at the wall.
- Free convection is driven only by buoyancy; induced velocities are much smaller.
- Smaller velocity → lower Re or Gr → lower Nu.
- Since $h = Nu\, k/L$, lower Nu gives lower h.

- **Q.** Explain mean film temperature and bulk mean temperature. **(SUMMER 2026, 4 Marks)**

Answer:

Mean film temperature:

$$\boxed{T_f = \frac{T_s + T_\infty}{2}}$$

Used to evaluate fluid properties near the surface in external convection.

Bulk mean temperature:

$$\boxed{T_m = \frac{T_{in} + T_{out}}{2}}$$

Used for internal pipe/duct flow property evaluation.

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2. Boundary Layer – Equations & Numerical:

- **Q.** Derive the two dimensional energy equation for thermal boundary layer over a flat plate. **(WINTER 2025, 7 Marks)**

Answer:

Steps:

1. Start from 2D steady incompressible energy equation with constant properties and no heat generation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)$$

2. For a boundary layer:

$$x \sim L, \quad y \sim \delta_t, \quad \delta_t \ll L$$

Therefore:

$$\frac{\partial^2 T}{\partial x^2} \ll \frac{\partial^2 T}{\partial y^2}$$

3. Neglect axial conduction:

$$\boxed{u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}}$$

Boundary conditions:

$$T(x,0) = T_s, \quad T(x,\infty) = T_\infty, \quad T(0,y) = T_\infty$$

- **Q.** The velocity distribution in the boundary layer is given by $\bar{U} = u/\delta$, where u is the velocity at a distance y from the plate and u = U at y = δ, δ being boundary layer thickness. Find: (i) The displacement thickness, (ii) The momentum thickness, (iii) The energy thickness, and (iv) The value of … **(SUMMER 2026)**

Answer:

- **Given:** Assume linear velocity profile:

$$\frac{u}{U} = \frac{y}{\delta}, \quad 0 \le y \le \delta$$

To find: Displacement thickness δ^*, momentum thickness θ, energy thickness δ_E, and shape factor H.

Formulas:

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy$$

$$\theta = \int_0^\delta \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

$$\delta_E = \int_0^\delta \frac{u}{U}\left[1 - \left(\frac{u}{U}\right)^2\right] dy$$

$$H = \frac{\delta^*}{\theta}$$

Solution:

Displacement thickness:

$$\delta^* = \int_0^\delta \left(1 - \frac{y}{\delta}\right) dy = \left[y - \frac{y^2}{2\delta}\right]_0^\delta = \delta - \frac{\delta}{2}$$

$$\boxed{\delta^* = \frac{\delta}{2}}$$

Momentum thickness:

$$\theta = \int_0^{\delta} \frac{y}{\delta}\left(1 - \frac{y}{\delta}\right) dy = \int_0^{\delta} \left(\frac{y}{\delta} - \frac{y^2}{\delta^2}\right) dy$$

$$= \frac{\delta}{2} - \frac{\delta}{3}$$

$$\boxed{\theta = \frac{\delta}{6}}$$

Energy thickness:

$$\delta_E = \int_0^{\delta} \frac{y}{\delta}\left[1 - \frac{y^2}{\delta^2}\right] dy = \int_0^{\delta} \left(\frac{y}{\delta} - \frac{y^3}{\delta^3}\right) dy$$

$$= \frac{\delta}{2} - \frac{\delta}{4}$$

$$\boxed{\delta_E = \frac{\delta}{4}}$$

Shape factor:

$$H = \frac{\delta^*}{\theta} = \frac{\delta/2}{\delta/6}$$

$$\boxed{H = 3}$$

Final Answer:

$$\boxed{\delta^* = \delta/2, \quad \theta = \delta/6, \quad \delta_E = \delta/4, \quad H = 3}$$
