## Chapter 6: Heat Exchangers

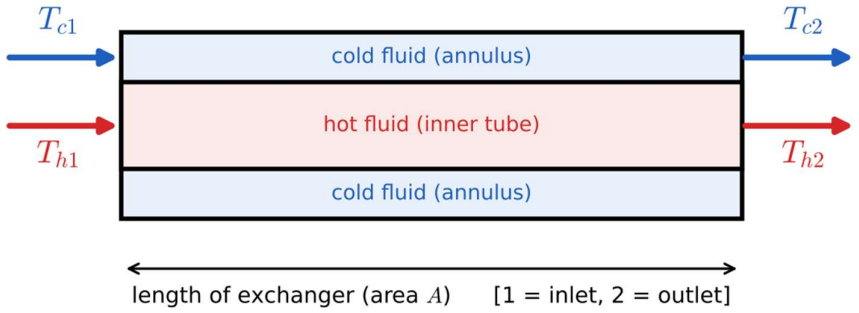
### Repeated Questions:

**1. LMTD:**

- **Q.** Derive LMTD for Parallel Flow/Counter Flow Heat Exchanger. **(WINTER 2022, 7 Marks; WINTER 2024, 7 Marks; WINTER 2025, 7 Marks)**

**Answer:**

(a) Parallel flow




(b) Counter flow

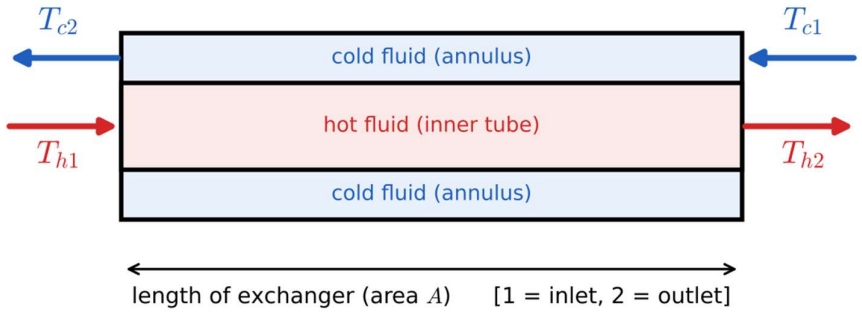



*Fig. 12: Parallel-flow and counter-flow concentric-tube heat exchangers with terminal temperatures*

- **Assumptions:** Steady flow, constant specific heats, constant overall heat transfer coefficient $U$, negligible heat loss to surroundings, no phase change.

**Steps:**

1. Consider a differential area $dA$. Heat transfer:

$$dQ = U\,dA\,(T_h - T_c) = U\,dA\,\Delta T$$

2. Energy balance on hot and cold fluids:

$$dQ = -m_h c_{ph}\,dT_h = m_c c_{pc}\,dT_c$$

Let $C_h = m_h c_{ph}$, $C_c = m_c c_{pc}$.

3. Change in temperature difference:

$$d(\Delta T) = dT_h - dT_c = -dQ\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

4. Substitute $dQ = U\,dA\,\Delta T$:

$$\frac{d(\Delta T)}{\Delta T} = -U\left(\frac{1}{C_h} + \frac{1}{C_c}\right)dA$$

5. Integrate from $A = 0$ to $A$:

$$\ln\frac{\Delta T_2}{\Delta T_1} = -UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

6. From total energy balance:

$$Q = C_h(T_{h1} - T_{h2}) = C_c(T_{c2} - T_{c1})$$

$$\frac{1}{C_h} + \frac{1}{C_c} = \frac{\Delta T_1 - \Delta T_2}{Q}$$

7. Substitute and rearrange:

$$Q = UA\,\frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

$$\boxed{Q = UA\,LMTD}$$

where

$$\boxed{LMTD = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}}$$

**For parallel flow:**

$$\Delta T_1 = T_{h1} - T_{c1}, \quad \Delta T_2 = T_{h2} - T_{c2}$$

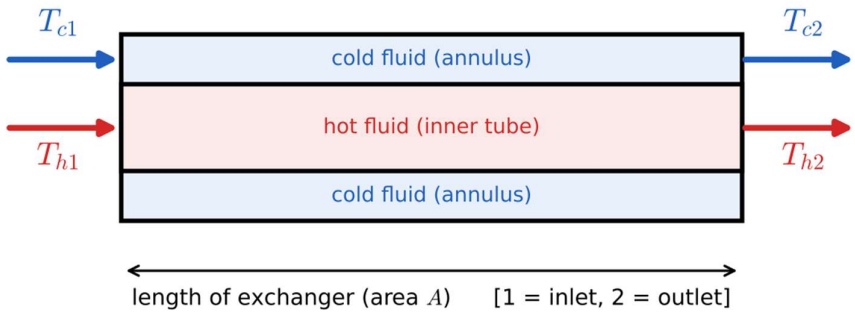
**For counter flow:**

$$\Delta T_1 = T_{h1} - T_{c2}, \quad \Delta T_2 = T_{h2} - T_{c1}$$

- **Q.** Derive an expression for LMTD in case of parallel flow heat exchanger. **(WINTER 2025, 7 Marks)**

**Answer:**

(a) Parallel flow




(b) Counter flow

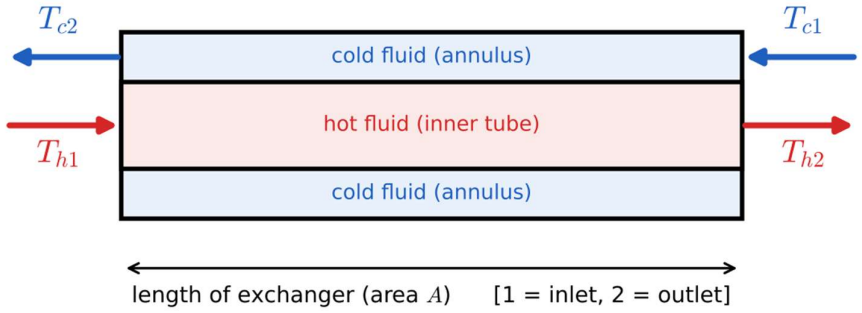



*Fig. 13: Parallel-flow and counter-flow concentric-tube heat exchangers with terminal temperatures*

- **Assumptions:** Steady flow, constant specific heats, constant overall heat transfer coefficient $U$, negligible heat loss to surroundings, no phase change.

**Steps:**

1. Consider a differential area $dA$. Heat transfer:

$$dQ = U\,dA\,(T_h - T_c) = U\,dA\,\Delta T$$

2. Energy balance on hot and cold fluids:

$$dQ = -m_h c_{ph}\,dT_h = m_c c_{pc}\,dT_c$$

Let $C_h = m_h c_{ph}$, $C_c = m_c c_{pc}$.

3. Change in temperature difference:

$$d(\Delta T) = dT_h - dT_c = -dQ\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

4. Substitute $dQ = U\,dA\,\Delta T$:

$$\frac{d(\Delta T)}{\Delta T} = -U\left(\frac{1}{C_h} + \frac{1}{C_c}\right)dA$$

5. Integrate from $A = 0$ to $A$:

$$\ln\frac{\Delta T_2}{\Delta T_1} = -UA\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

6. From total energy balance:

$$Q = C_h(T_{h1} - T_{h2}) = C_c(T_{c2} - T_{c1})$$

$$\frac{1}{C_h} + \frac{1}{C_c} = \frac{\Delta T_1 - \Delta T_2}{Q}$$

7. Substitute and rearrange:

$$Q = UA\,\frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

$$\boxed{Q = UA\,LMTD}$$

where

$$\boxed{LMTD = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}}$$

**For parallel flow:**

$$\Delta T_1 = T_{h1} - T_{c1}, \quad \Delta T_2 = T_{h2} - T_{c2}$$

**For counter flow:**

$$\Delta T_1 = T_{h1} - T_{c2}, \quad \Delta T_2 = T_{h2} - T_{c1}$$

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**2. Effectiveness & NTU:**

- **Q.** Define and Explain: Effectiveness, NTU, Heat Capacity Ratio. **(SUMMER 2022, 7 Marks; WINTER 2023, 7 Marks OR; SUMMER 2024, 7 Marks)**

**Answer:**

**Effectiveness:**

$$\boxed{\varepsilon = \frac{Q_{actual}}{Q_{max}}}$$

where

$$Q_{max} = C_{min}\left(T_{h,in} - T_{c,in}\right)$$

and $C_{min} = \min\left(m_h c_{ph}, m_c c_{pc}\right)$.

**NTU (Number of Transfer Units):**

$$\boxed{NTU = \frac{UA}{C_{min}}}$$

It measures the size of the heat exchanger relative to the minimum heat capacity rate.

**Heat capacity ratio:**

$$\boxed{C_r = \frac{C_{min}}{C_{max}}}$$

where $C_{max} = \max\left(m_h c_{ph}, m_c c_{pc}\right)$.

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- **Q.** Derive the expression for Effectiveness (ε) for Parallel Flow. **(SUMMER 2022, 7 Marks; WINTER 2023, 7 Marks OR; SUMMER 2025, 7 Marks)**

**Answer:**

**Steps:**

1. For parallel flow, LMTD:

$$Q = UA\,\frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

with

$$\Delta T_1 = T_{h,in} - T_{c,in}, \quad \Delta T_2 = T_{h,out} - T_{c,out}$$

2. Let $C_{min} = C_h$ (without loss of generality). Then:

$$Q = \varepsilon C_{min} \Delta T_1$$

$$T_{h,out} = T_{h,in} - \frac{Q}{C_h}$$

$$T_{c,out} = T_{c,in} + \frac{Q}{C_c}$$

3. Express $\Delta T_2$:

$$\Delta T_2 = T_{h,out} - T_{c,out} = \Delta T_1 - Q\left(\frac{1}{C_h} + \frac{1}{C_c}\right)$$

$$\frac{\Delta T_2}{\Delta T_1} = 1 - \varepsilon\left(1 + \frac{C_{min}}{C_{max}}\right) = 1 - \varepsilon(1 + C_r)$$

4. Substitute in LMTD expression:

$$Q = UA\,\frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$

$$\varepsilon C_{min} \Delta T_1 = UA\,\frac{\Delta T_1\left[1 - \left(1 - \varepsilon(1 + C_r)\right)\right]}{-\ln[1 - \varepsilon(1 + C_r)]}$$

5. Using $NTU = UA/C_{min}$ and simplifying:

$$\boxed{\varepsilon = \frac{1 - \exp[-NTU(1 + C_r)]}{1 + C_r}}$$

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**3. Classification of Heat Exchangers:**

- **Q.** Classification of Heat Exchangers and their Applications. **(SUMMER 2023, 4 Marks; WINTER 2024, 7 Marks; SUMMER 2025, 4 Marks; SUMMER 2026, 4 Marks)**

**Answer:**

| Basis | Types | Applications |
|---|---|---|
| Transfer process | Direct contact, Indirect contact | Cooling towers; shell-and-tube boilers |
| Flow arrangement | Parallel, Counter, Cross, Shell-and-tube | Oil coolers; condensers; radiators |
| Construction | Tubular, Plate, Extended surface | Power plants; food processing; air conditioning |
| Heat transfer mechanism | Single phase, Phase change | Economizers; evaporators and condensers |

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- **Q.** Classify Heat Exchangers. **(SUMMER 2026, 4 Marks)**

**Answer:**

| Basis | Types | Applications |
|---|---|---|
| Transfer process | Direct contact, Indirect contact | Cooling towers; shell-and-tube boilers |
| Flow arrangement | Parallel, Counter, Cross, Shell-and-tube | Oil coolers; condensers; radiators |
| Construction | Tubular, Plate, Extended surface | Power plants; food processing; air conditioning |
| Heat transfer mechanism | Single phase, Phase change | Economizers; evaporators and condensers |

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**4. Fouling Factor:**

- **Q.** Fouling Factor: Definition and Causes. **(WINTER 2022, 3 Marks; SUMMER 2023, 3 Marks; SUMMER 2024, 3 Marks)**

**Answer:**

- **Definition:** Fouling factor $R_f$ is the additional thermal resistance developed on heat transfer surfaces due to deposition of unwanted material.

$$\boxed{R_f = \frac{1}{U_{dirty}} - \frac{1}{U_{clean}}}$$

Units: $m^2K/W$.

**Causes:**

- Scale formation from dissolved salts.
- Corrosion products and rust.
- Biological growth (algae, bacteria).
- Deposition of suspended solids and dirt.
- Chemical reaction products and coking.

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**5. Temperature Distribution in Heat Exchangers:**

- **Q.** Draw temperature distribution for Condenser and Evaporator. **(SUMMER 2022; WINTER 2024, 4 Marks; SUMMER 2026, 4 Marks OR)**

**Answer:**

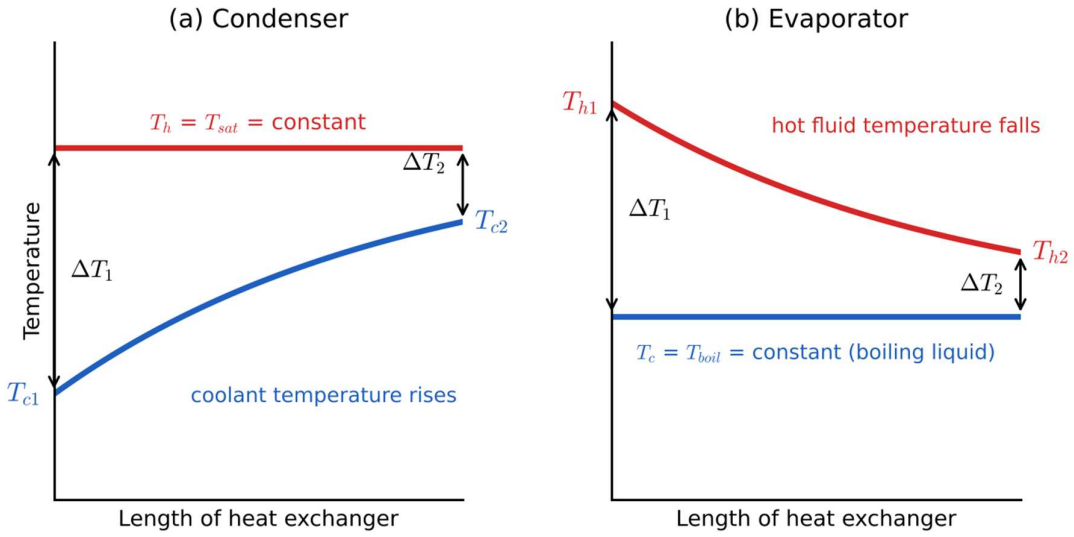



*Fig. 14: Temperature distribution along the length of a condenser and an evaporator*

- **Condenser:** Hot fluid remains at constant temperature during phase change; cold fluid temperature increases.
- **Evaporator:** Cold fluid remains at constant boiling temperature; hot fluid temperature decreases.

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- **Q.** Show the temperature variation along length of heat exchanger when: 1. Hot and cold fluids flow in parallel and counter flow. 2. Steam condenses on outside of condenser tube with water flowing inside the tube as coolant. 3. Hot fluid as used for evaporating another liquid. **(SUMMER 2026, 4 Marks OR)**

**Answer:**

[Part 1 (temperature variation for parallel and counter flow) – answer not found in the answer chat – to be added]

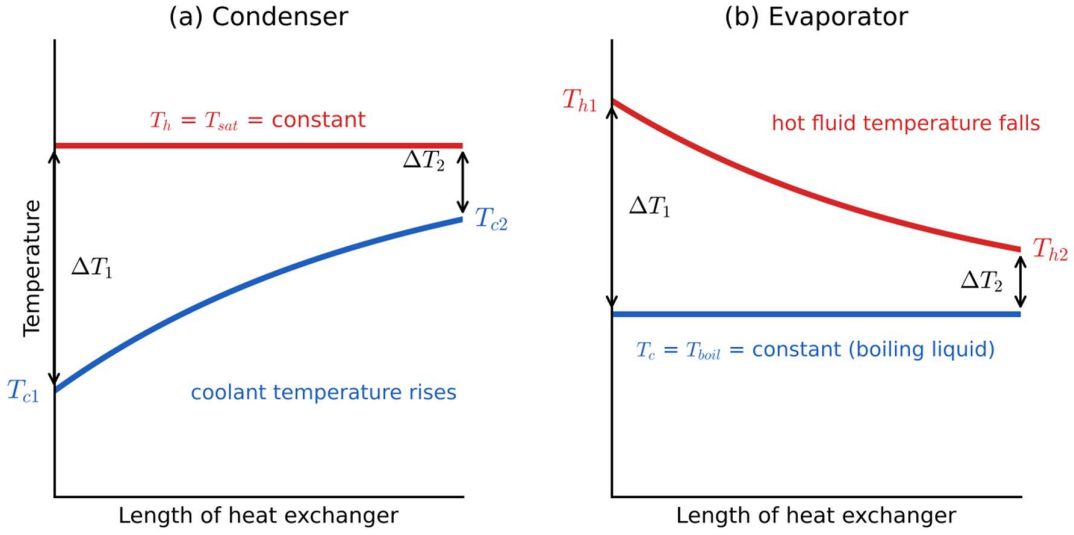



*Fig. 15: Temperature distribution along the length of a condenser and an evaporator*

- **Condenser:** Hot fluid remains at constant temperature during phase change; cold fluid temperature increases.
- **Evaporator:** Cold fluid remains at constant boiling temperature; hot fluid temperature decreases.

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## Other Important Questions:

**1. Heat Exchanger Numericals:**

- **Q.** Numericals on Heat Exchanger sizing (finding Area/Number of Tubes). **(WINTER 2023, 7 Marks)**

**Answer:**

**Given:**

- Oil: $m_h = 2.78\ kg/s$, $c_{ph} = 2.095\ kJ/kgK = 2095\ J/kgK$
- Oil inlet $T_{h,in} = 80°C$, outlet $T_{h,out} = 50°C$
- Water: $m_c = 2.22\ kg/s$, $c_{pc} = 4.18\ kJ/kgK = 4180\ J/kgK$
- Water inlet $T_{c,in} = 25°C$
- Overall heat transfer coefficient $U = 300\ W/m^2K$

To find: Area $A$

**Formula:**

$$Q = m_h c_{ph}(T_{h,in} - T_{h,out})$$
$$Q = m_c c_{pc}(T_{c,out} - T_{c,in})$$
$$LMTD = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$
$$A = \frac{Q}{U\ LMTD}$$

**Solution:**

**Heat transfer rate:**

$$Q = 2.78(2095)(80 - 50) = 174723\ W$$

**Water outlet temperature:**

$$174723 = 2.22(4180)(T_{c,out} - 25)$$
$$T_{c,out} = 25 + \frac{174723}{9279.6} = 43.83°C$$

**Counter flow temperature differences:**

$$\Delta T_1 = T_{h,in} - T_{c,out} = 80 - 43.83 = 36.17\ K$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 50 - 25 = 25\ K$$
$$LMTD = \frac{36.17 - 25}{\ln(36.17/25)} = 30.25\ K$$

**Area:**

$$A = \frac{174723}{300(30.25)} = 19.25\ m^2$$

**Final Answer:**

$$\boxed{A \approx 19.25\ m^2}$$

**Given:**

- Oil: $m_h = 1400\ kg/hr = 0.3889\ kg/s$, $c_{ph} = 3\ kJ/kgK = 3000\ J/kgK$
- Oil inlet $T_{h,in} = 100°C$, outlet $T_{h,out} = 30°C$
- Water: $m_c = 1300\ kg/hr = 0.3611\ kg/s$, $T_{c,in} = 20°C$, $c_{pc} = 4.18\ kJ/kgK = 4180\ J/kgK$

- Overall heat transfer coefficient $U = 3975\ kJ/m^2hrK = 1104.17\ W/m^2K$

To find: Area $A$

**Formula:**

$$Q = m_h c_{ph}(T_{h,in} - T_{h,out})$$
$$Q = m_c c_{pc}(T_{c,out} - T_{c,in})$$
$$LMTD = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$
$$A = \frac{Q}{U\ LMTD}$$

**Solution:**

**Heat transfer rate:**

$$Q = 0.3889(3000)(100 - 30) = 81667\ W$$

**Water outlet temperature:**

$$81667 = 0.3611(4180)(T_{c,out} - 20)$$
$$T_{c,out} = 20 + \frac{81667}{1509.4} = 74.10°C$$

**Counter flow temperature differences:**

$$\Delta T_1 = T_{h,in} - T_{c,out} = 100 - 74.10 = 25.90\ K$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 30 - 20 = 10\ K$$
$$LMTD = \frac{25.90 - 10}{\ln(25.90/10)} = 16.71\ K$$

**Area:**

$$A = \frac{81667}{1104.17(16.71)} = 4.43\ m^2$$

**Final Answer:**

$$\boxed{A \approx 4.43\ m^2}$$

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- **Q.** In a counter flow heat exchanger, oil (cp = 2.095 kJ/kg-K) at the rate of 2.78 kg/sec is cooled from 80 °C to 50 °C by 2.22 kg/sec of water entering at 25 °C. Determine the area of heat exchanger. Take U = 300 W/m²K and Cp = 4.18 kJ/kgK. **(WINTER 2025, 7 Marks OR)**

**Answer:**

**Given:**

- Oil: $m_h = 2.78\ kg/s$, $c_{ph} = 2.095\ kJ/kgK = 2095\ J/kgK$
- Oil inlet $T_{h,in} = 80°C$, outlet $T_{h,out} = 50°C$
- Water: $m_c = 2.22\ kg/s$, $c_{pc} = 4.18\ kJ/kgK = 4180\ J/kgK$
- Water inlet $T_{c,in} = 25°C$
- Overall heat transfer coefficient $U = 300\ W/m^2K$

To find: Area $A$

**Formula:**

$$Q = m_h c_{ph}(T_{h,in} - T_{h,out})$$
$$Q = m_c c_{pc}(T_{c,out} - T_{c,in})$$
$$LMTD = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$
$$A = \frac{Q}{U\ LMTD}$$

**Solution:**

**Heat transfer rate:**

$$Q = 2.78(2095)(80 - 50) = 174723\ W$$

**Water outlet temperature:**

$$174723 = 2.22(4180)\left(T_{c,out} - 25\right)$$

$$T_{c,out} = 25 + \frac{174723}{9279.6} = 43.83°C$$

**Counter flow temperature differences:**

$$\Delta T_1 = T_{h,in} - T_{c,out} = 80 - 43.83 = 36.17\ K$$

$$\Delta T_2 = T_{h,out} - T_{c,in} = 50 - 25 = 25\ K$$

$$LMTD = \frac{36.17 - 25}{\ln(36.17/25)} = 30.25\ K$$

**Area:**

$$A = \frac{174723}{300(30.25)} = 19.25\ m^2$$

**Final Answer:**

$$\boxed{A \approx 19.25\ m^2}$$

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- **Q.** Hot water having specific heat 4200 J/kgK flows through a heat exchanger at the rate of 4 kg/min with an inlet temperature of 100 °C. A cold fluid having a specific heat 2400 J/kgK flows in at a rate of 8 kg/min and with inlet temperature 20 °C. Make calculations for the maximum possible effectiveness if the fluid flow conforms to a) parallel flow arrangement b) counter flow arrangement. **(SUMMER 2026)**

**Answer:**

**Given:**

- Hot water: $m_h = 4\ kg/min = 0.06667\ kg/s, c_{ph} = 4200\ J/kgK, T_{h,in} = 100°C$
- Cold fluid: $m_c = 8\ kg/min = 0.13333\ kg/s, c_{pc} = 2400\ J/kgK, T_{c,in} = 20°C$

To find: Maximum possible effectiveness for (a) parallel flow, (b) counter flow.

**Formula:**

$$C_h = m_h c_{ph}, \quad C_c = m_c c_{pc}$$

$$C_{min} = \min(C_h, C_c), \quad C_r = \frac{C_{min}}{C_{max}}$$

**Parallel flow maximum (infinite area):**

$$\varepsilon_{max,parallel} = \frac{1}{1 + C_r}$$

**Counter flow maximum (infinite area, $C_r < 1$):**

$$\varepsilon_{max,counter} = 1$$

**Solution:**

**Heat capacity rates:**

$$C_h = 0.06667(4200) = 280\ W/K$$

$$C_c = 0.13333(2400) = 320\ W/K$$

$$C_{min} = 280\ W/K, \quad C_{max} = 320\ W/K$$

$$C_r = \frac{280}{320} = 0.875$$

**Parallel flow maximum effectiveness:**

$$\varepsilon_{max,parallel} = \frac{1}{1 + 0.875} = 0.533$$

**Counter flow maximum effectiveness:**

$$\varepsilon_{max,counter} = 1.0$$

**Final Answer:**

$$\boxed{\varepsilon_{max,parallel} \approx 0.533\ (53.3\%), \quad \varepsilon_{max,counter} = 1.0\ (100\%)}$$

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- **Q.** In a counter flow heat exchanger, oil (cp = 3 kJ/kg-K) at the rate of 1400 kg/hr is cooled from 100 °C to 30 °C by water that enters the exchanger at 20 °C at the rate of 1300 kg/hr. Determine the heat exchanger area for an overall heat transfer coefficient of 3975 kJ/m²-hr-K. **(SUMMER 2026 OR)**

**Answer:**

**Given:**

- Oil: $m_h = 1400\ kg/hr = 0.3889\ kg/s$, $c_{ph} = 3\ kJ/kgK = 3000\ J/kgK$
- Oil inlet $T_{h,in} = 100°C$, outlet $T_{h,out} = 30°C$
- Water: $m_c = 1300\ kg/hr = 0.3611\ kg/s$, $T_{c,in} = 20°C$, $c_{pc} = 4.18\ kJ/kgK = 4180\ J/kgK$
- Overall heat transfer coefficient $U = 3975\ kJ/m^2hrK = 1104.17\ W/m^2K$

To find: Area $A$

**Formula:**

$$Q = m_h c_{ph}(T_{h,in} - T_{h,out})$$
$$Q = m_c c_{pc}(T_{c,out} - T_{c,in})$$
$$LMTD = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$
$$A = \frac{Q}{U\ LMTD}$$

**Solution:**

**Heat transfer rate:**

$$Q = 0.3889(3000)(100 - 30) = 81667\ W$$

**Water outlet temperature:**

$$81667 = 0.3611(4180)(T_{c,out} - 20)$$
$$T_{c,out} = 20 + \frac{81667}{1509.4} = 74.10°C$$

**Counter flow temperature differences:**

$$\Delta T_1 = T_{h,in} - T_{c,out} = 100 - 74.10 = 25.90\ K$$
$$\Delta T_2 = T_{h,out} - T_{c,in} = 30 - 20 = 10\ K$$
$$LMTD = \frac{25.90 - 10}{\ln(25.90/10)} = 16.71\ K$$

**Area:**

$$A = \frac{81667}{1104.17(16.71)} = 4.43\ m^2$$

**Final Answer:**

$$\boxed{A \approx 4.43\ m^2}$$

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