GUJARAT TECHNOLOGICAL UNIVERSITY

BE-5 SEMESTER – OLD PAPER – S22 TO S26 – ANSWER BANK

Subject Name & Code: Dynamics of Machinery - 3151911

Chapter 1: Dynamic Force Analysis of Mechanisms

Repeated Questions:

1. State and Explain D'Alembert's Principle:

- **Q.** State and explain D'Alembert's Principle. **(SUMMER 2022, 4 Marks; WINTER 2023, 3 Marks; SUMMER 2025, 3 Marks; SUMMER 2026, 3 Marks)**

Answer:

- **D'Alembert's Principle:** A dynamic system can be analysed as a static system by adding a fictitious inertia force equal and opposite to the product of mass and acceleration.

For a body of mass m having acceleration a, inertia force is

$$F_i = -ma$$

For rotation with angular acceleration α, inertia couple is

$$T_i = -I\alpha$$

Dynamic equilibrium equation:

$$\Sigma F + F_i = 0, \ \Sigma T + T_i = 0$$

- **Significance:** Converts a dynamics problem into an equivalent statics problem; used for force analysis of slider-crank, IC engine, and reciprocating mechanisms.

---

2. Analytical Method for Slider-Crank Mechanism (Numerical):

- **Q.** In a slider crank mechanism, the length of the crank and connecting rod are 150 mm and 600 mm respectively. The crank position is 60° from IDC. The crank shaft speed is 450 rpm. Using analytical method, determine: (i) Velocity and acceleration of the slider, (ii) Angular velocity and angular acceleration of the connecting rod. **(SUMMER 2022, 7 Marks OR; SUMMER 2023, 7 Marks OR)**

Answer:

Given:

- Crank radius, r = 150 mm = 0.15 m
- Connecting rod length, l = 600 mm = 0.60 m
- Obliquity ratio, n = l/r = 4
- Crank angle from IDC, θ = 60°
- Speed, N = 450 rpm

$$\omega = 2\pi N/60 = 2\pi(450)/60 = 47.124 \text{ rad/s}$$

- **To Find:** v, a of slider; ω_c, α_c of connecting rod.

Formulae:

$$v = r\omega\,(\sin\theta + \sin2\theta / 2n)$$
$$a = r\omega^2\,(\cos\theta + \cos2\theta / n)$$
$$\omega_c = \omega\cos\theta / \sqrt{(n^2 - \sin^2\theta)}$$
$$\alpha_c = -\omega^2\sin\theta\,(n^2 - 1) / (n^2 - \sin^2\theta)\text{^}(3/2)$$

Solution:

$$\sin60° = 0.8660, \sin120° = 0.8660, \cos60° = 0.5, \cos120° = -0.5$$
$$v = 0.15(47.124)(0.8660 + 0.8660/(2\times4))$$
$$v = 7.0686(0.8660 + 0.1083) = 6.887 \text{ m/s}$$

$a = 0.15(47.124)^2\ (0.5 + (-0.5)/4)$

$a = 333.10(0.375) = 124.91\ m/s^2$

$\omega_c = 47.124(0.5)/\sqrt{(16 - 0.8660^2)} = 23.562/3.905 = 6.034\ rad/s$

$\alpha_c = -(47.124)^2(0.8660)(16 - 1)/(16 - 0.8660^2)^{\wedge}(3/2)$

$\alpha_c = -28846/59.55 = -484.4\ rad/s^2$

Final Answer:

$v = 6.89\ m/s,\ a = 124.91\ m/s^2$

$\omega_c = 6.03\ rad/s,\ \alpha_c = 484.4\ rad/s^2$ (opposite to rod rotation)

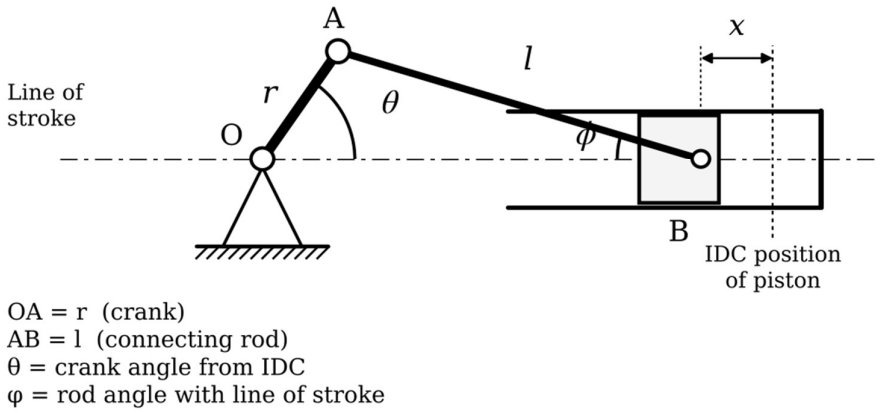


Fig. 1: Slider-crank mechanism

---

3. Forces in IC Engine / Steam Engine Mechanism (Numerical):

- **Q.** Find (i) thrust on connecting rod, (ii) piston side thrust, (iii) torque acting on the crank shaft, (iv) load on main bearings when crank is at a given angle. (Variants exist for steam engines.) **(WINTER 2023, 7 Marks; WINTER 2024, 7 Marks)**

Answer:

Given (symbols):

Gas/steam pressure force on piston P_g, reciprocating mass m_rec, rotating mass m_rot, crank radius r, connecting rod length l, speed N, crank angle θ.

To Find:

(i) Thrust on connecting rod F_c

(ii) Piston side thrust F_N

(iii) Torque on crankshaft T

(iv) Load on main bearings F_b

Formulae:

$n = l/r,\ \varphi = \sin^{-1}(\sin\theta / n)$

$a = r\omega^2\ (\cos\theta + \cos 2\theta / n)$

$F_i = m_rec\ a$

$F_p = P_g - F_i$ (net piston effort)

$F_c = F_p / \cos\varphi$

$F_N = F_p \tan\varphi$

$T = F_p\, r\, (\sin\theta + \sin 2\theta / 2n)$

$F_b = \sqrt{[(F_c \cos(\theta+\varphi) \pm m_{rot}\, \omega^2\, r)^2 + (F_c \sin(\theta+\varphi))^2]}$

Solution:

1. Calculate $\omega = 2\pi N/60$.
2. Calculate a, then inertia force $F_i = m_{rec}\, a$.
3. Find net piston effort $F_p = P_g - F_i$.
4. Resolve F_p along connecting rod and perpendicular to cylinder.
5. Calculate torque T and main bearing reaction F_b.
6. For two main bearings, load per bearing = $F_b/2$.

Final Answer:

Substitute the given numerical data in the above equations.

$F_c = F_p / \cos\varphi,\ F_N = F_p \tan\varphi,\ T = F_p\, r\, (\sin\theta + \sin 2\theta / 2n)$

$F_b = \sqrt{[(F_c \cos(\theta+\varphi) \pm m_{rot}\, \omega^2\, r)^2 + (F_c \sin(\theta+\varphi))^2]}$

Net piston effort $F_p = P_g - F_i$

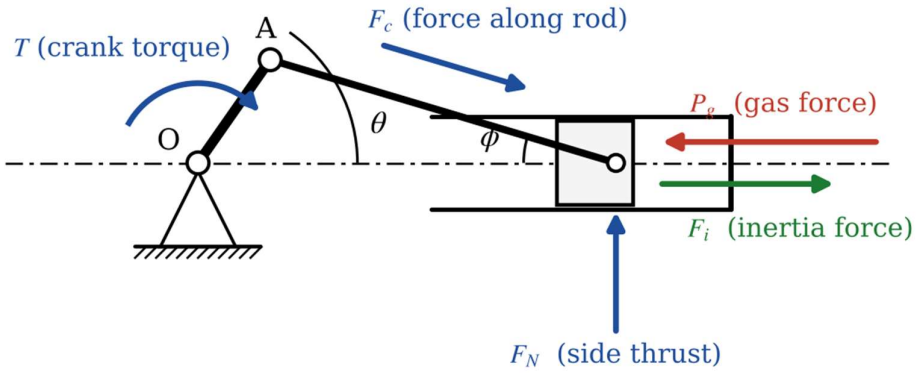


Fig. 2: Slider-crank mechanism with gas force, inertia force, rod force, side thrust and crank torque

---

4. Inertia Force and Inertia Couple:

- **Q.** Define Inertia Force and Inertia Couple. **(SUMMER 2022, 3 Marks; WINTER 2022, 3 Marks)**

Answer:

- **Inertia Force:** A fictitious force equal and opposite to the product of mass and acceleration.

  $F_i = -ma$
- **Inertia Couple:** A fictitious couple equal and opposite to the product of mass moment of inertia and angular acceleration.

  $T_i = -I\alpha$
- It is used in D'Alembert's principle.
- It converts dynamic equilibrium into static equilibrium.
- Direction is always opposite to acceleration/angular acceleration.

Other Important Questions:

1. Dynamically Equivalent System:

- **Q.** Explain the Dynamical Equivalent System of two masses. **(SUMMER 2024, 4 Marks)**

Answer:

- **Definition:** A dynamically equivalent system is a substitute system of two masses that has the same total mass, same centre of gravity, and same mass moment of inertia as the original rigid body.

For two masses m_1 and m_2 at distances l_1 and l_2 from CG:

$$m_1 + m_2 = m$$
$$m_1 l_1 = m_2 l_2$$
$$I = m_1 l_1^2 + m_2 l_2^2$$
$$m_1 = m\, l_2/(l_1 + l_2),\ m_2 = m\, l_1/(l_1 + l_2)$$

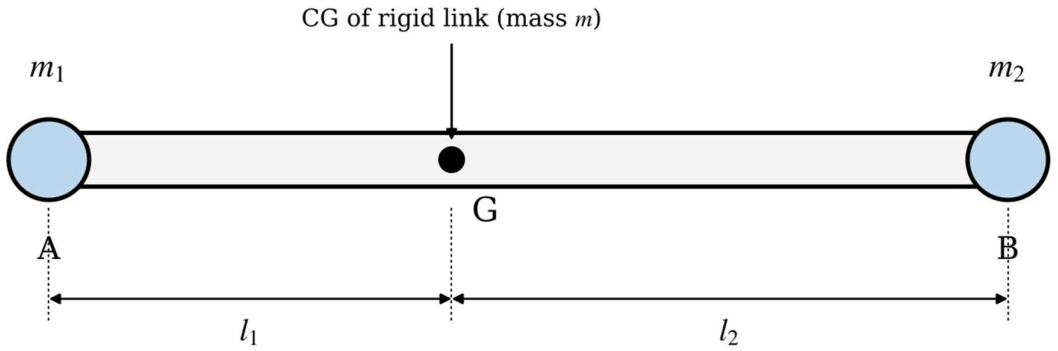


Fig. 3: Dynamically equivalent system of a rigid link (substitute masses m_1 and m_2)

- **Q.** What are the requirements of an equivalent dynamical system? **(WINTER 2022, 3 Marks)**

Answer:

- Total mass of substitute system = total mass of original body.
- Centre of gravity of substitute system = CG of original body.
- Mass moment of inertia about CG = mass moment of inertia of original body.
- It must produce the same dynamic force and couple for the same motion.

---

2. Inertia Torque Calculation (Numerical):

- **Q.** Determine the inertia torque on the crankshaft when the crank has turned 40° from IDC. (Data for reciprocating parts and connecting rod inertia is given.) **(SUMMER 2025, 7 Marks)**

Answer:

Given (symbols):

m_rec, r, l, N, θ = 40°, connecting rod inertia I_rod.

- **To Find:** Inertia torque on crankshaft T_i.

Formulae:

$$n = l/r,\ \omega = 2\pi N/60$$
$$a = r\omega^2 (\cos\theta + \cos 2\theta / n)$$
$$F_i = m_rec\ a$$
$$T_rec = F_i\, r\, (\sin\theta + \sin 2\theta / 2n)$$
$$\alpha_rod = -\omega^2 \sin\theta\, (n^2 - 1) / (n^2 - \sin^2\theta)\text{^}(3/2)$$

$T_{rod} = I_{rod}\,\alpha_{rod}$

$T_i = T_{rec} + T_{rod}$

Solution:

1. Calculate ω, n, and acceleration a.
2. Find inertia force $F_i = m_{rec}\,a$.
3. Find inertia torque due to reciprocating mass T_rec.
4. Find angular acceleration of connecting rod α_rod and T_rod = I_rod α_rod.
5. Add algebraically with proper sign to get total inertia torque.

Final Answer:

$T_i = m_{rec}\,a\,r\,(\sin\theta + \sin 2\theta / 2n) + I_{rod}\,\alpha_{rod}$

---

3. Piston Effort and Force along the Connecting Rod:

- **Q.** Analyze forces like Piston effort and force acting along the connecting rod. **(SUMMER 2024, 7 Marks)**

Answer:

- **Piston Effort:** Net force acting on piston along line of stroke.

$F_p = P_g - F_i$

where P_g = gas/steam force, $F_i = m_{rec}\,a$ = inertia force.

- **Force along Connecting Rod:** Resolving piston effort along connecting rod:

$F_c = F_p / \cos\varphi$

where $\phi = \sin^{-1}(\sin\theta / n)$.

- **Piston Side Thrust:**

$F_N = F_p \tan\varphi$

- **Crank Torque:**

$T = F_p\,r\,(\sin\theta + \sin 2\theta / 2n)$

Steps:

1. Find acceleration of reciprocating parts.
2. Find inertia force F_i.
3. Find net piston effort F_p.
4. Resolve F_p along connecting rod and normal to cylinder.
5. Find crank torque from tangential component.

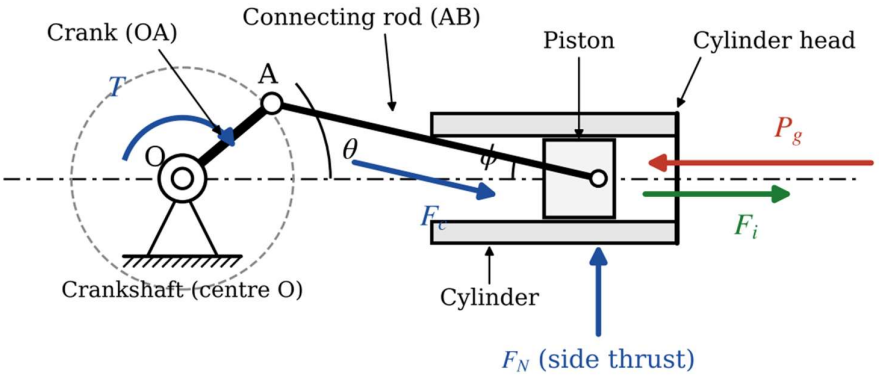


Fig. 4: Forces acting on piston, connecting rod and crank

4. Inertia Force in Slider-Crank Mechanism (Numerical):

- **Q.** Calculate the inertia force in a slider-crank mechanism given crank speed and dimensions. **(SUMMER 2026, 4 Marks)**

Answer:

Given (symbols):

m, r, l, N, θ.

- **To Find:** Inertia force F_i.

Formula:

$a = r\omega^2 (\cos\theta + \cos 2\theta / n),\ n = l/r$

$F_i = ma$

Solution:

1. Calculate ω = 2πN/60.
2. Calculate acceleration a using slider-crank formula.
3. Multiply by reciprocating mass m.
4. Inertia force acts opposite to acceleration.

Final Answer:

$F_i = m\ r\ \omega^2 (\cos\theta + \cos 2\theta / n)$, opposite to acceleration

---

Chapter 2: Turning Moment Diagrams and Flywheels

Repeated Questions:

1. Flywheel Design for Multi-cylinder Engine (Numerical):

- **Q.** The turning moment diagram for a multi-cylinder engine is given (scale and areas provided). If the total fluctuation of speed is not to exceed ±x% of the mean, find the necessary mass of the flywheel of radius R. **(SUMMER 2022, 7 Marks; WINTER 2022, 7 Marks; WINTER 2025, 7 Marks)**

Answer:

Given:

Turning moment diagram areas and scale; radius of flywheel R; mean speed N; total fluctuation of speed not to exceed ±x%.

$$\omega = 2\pi N/60,\ C_s = (N_1 - N_2)/N = 2x/100$$

- **To Find:** Necessary mass of flywheel m.

Formula:

$$\Delta E = I\omega^2 C_s,\ I = mR^2$$

$$m = \Delta E / (R^2\omega^2 C_s)$$

Solution:

1. Calculate net area of TMD = work done per cycle.
2. Draw mean torque line:

$$T_m = \text{Net work per cycle} / \text{angle per cycle}$$

3. Find maximum and minimum energy points from cumulative area.
4. Maximum fluctuation of energy:

$$\Delta E = E_max - E_min$$

5. For ±x% speed fluctuation:

$$C_s = 2x/100$$

6. Substitute in:

$$m = \Delta E / (R^2\omega^2 C_s)$$

Final Answer:

$$m = \Delta E / (R^2\omega^2 C_s)$$

---

2. Flywheel in Punching Press (Numerical):

- **Q.** A flywheel is attached to a punching press. Given max/min speed, energy per hole, punching time, etc., find (i) power required by the motor, and (ii) mass of the flywheel. **(WINTER 2023, 7 Marks OR; SUMMER 2025, 7 Marks)**

Answer:

Given:

Energy required per hole E; punching time t_p; cycle time t_c; maximum and minimum speeds N_1, N_2; flywheel radius R.

To Find:

(i) Power required by motor P

(ii) Mass of flywheel m

Formula:

$$P = E / t_c$$

$$\Delta E = E - P\ t_p$$

$$C_s = (N_1 - N_2)/N,\ N = (N_1 + N_2)/2$$

$$m = \Delta E / (R^2\omega^2 C_s)$$

Solution:

1. Motor supplies energy uniformly over the cycle:

   $P = E / t_c$

2. During punching, motor supplies P t_p. Flywheel supplies the remaining energy:

   $\Delta E = E - P\, t_p$

3. Calculate mean speed and coefficient of fluctuation of speed:

   $N = (N_1 + N_2)/2,\ C_s = (N_1 - N_2)/N$

4. Calculate ω = 2πN/60.

5. Find flywheel mass:

   $m = \Delta E / (R^2 \omega^2 C_s)$

Final Answer:

$P = E / t_c,\ m = (E - P\, t_p) / (R^2 \omega^2 C_s)$

---

3. Draw and Explain Turning Moment Diagrams:

- **Q.** Draw the turning moment diagram for a four-stroke IC engine / multi-cylinder engine. **(SUMMER 2022, 4 Marks; WINTER 2023, 4 Marks)**

Answer:

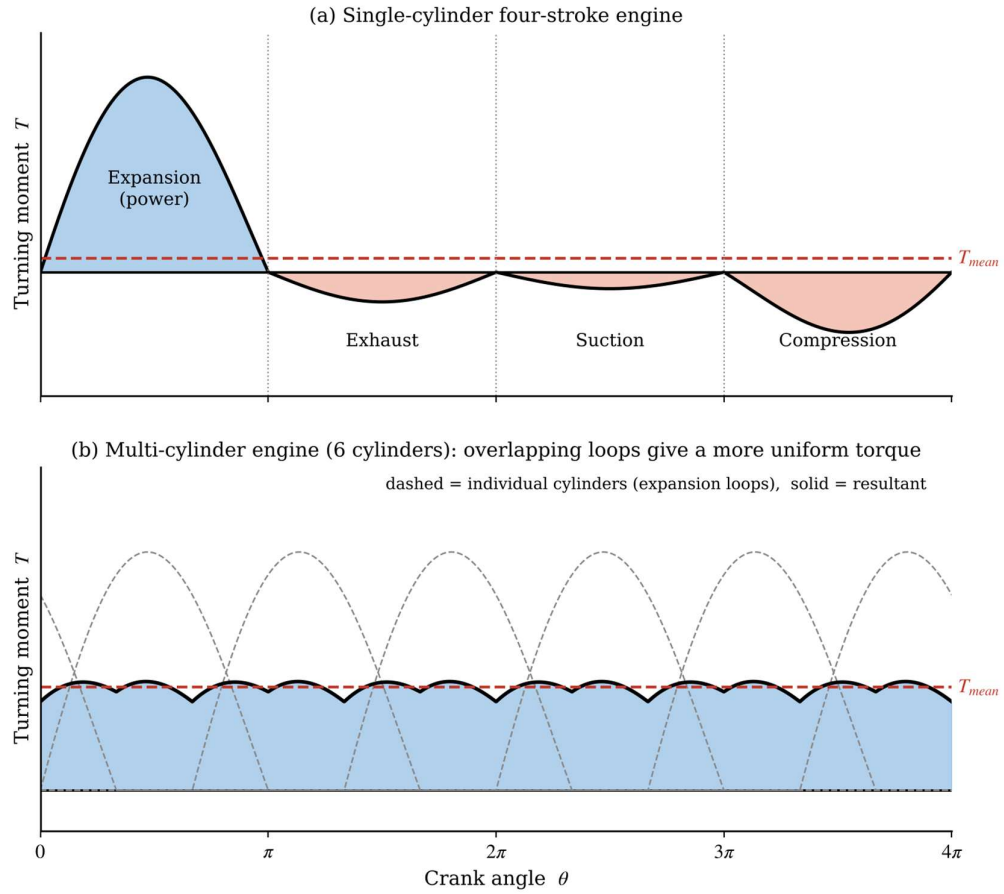


Fig. 5: Turning moment diagram: (a) single-cylinder four-stroke engine, (b) multi-cylinder engine

Explanation:

- **Four-stroke IC engine:** One working cycle takes two revolutions = 4π rad. Only expansion stroke produces positive torque; suction, compression, and exhaust consume energy.
- **Mean torque line:** The horizontal line represents average torque T_m.

- **Multi-cylinder engine:** Power strokes overlap, so torque fluctuations are smaller and flywheel size is reduced.
- **Flywheel function:** Stores energy when torque exceeds mean and supplies energy when torque is below mean.

---

4. Define Key Terms:

- **Q.** Define: Coefficient of Fluctuation of Energy, Coefficient of Fluctuation of Speed, Maximum Fluctuation of Energy. **(WINTER 2022, 3 Marks; WINTER 2023, 3 Marks; WINTER 2025, 4 Marks)**

Answer:

Term	Definition / Formula
Coefficient of fluctuation of energy	Ratio of maximum fluctuation of energy to mean kinetic energy: $C_e = \Delta E / E$
Coefficient of fluctuation of speed	Ratio of speed fluctuation to mean speed: $C_s = (N_1 - N_2)/N = (\omega_1 - \omega_2)/\omega$
Maximum fluctuation of energy	Difference between maximum and minimum energies: $\Delta E = E_{max} - E_{min} = I\omega^2 C_s$

Other Important Questions:

1. Role of a Flywheel:

- **Q.** Explain the role of a flywheel. How does it differ from a governor? **(SUMMER 2025, 4 Marks)**

Answer:

Role of flywheel:

- Stores energy during the period when driving torque exceeds resisting torque.
- Supplies stored energy when resisting torque exceeds driving torque.
- Reduces fluctuation of speed during each cycle.
- Makes the engine run smoothly and reduces required motor size in punching presses.

Flywheel	Governor
Controls cyclic speed fluctuation	Controls mean speed with load variation
Stores and releases energy	Regulates fuel/working fluid supply
Works continuously every cycle	Works only when load changes

---

2. Single-cylinder Engine Flywheel (Numerical):

- **Q.** A single cylinder, four-stroke gas engine develops X kW at Y rpm. The work done during expansion is Z times the work during compression. If the total fluctuation of speed is not to exceed ±4%, find the moment of inertia of the flywheel. **(WINTER 2022, 7 Marks)**

Answer:

Given:

Power developed P kW; speed N rpm; expansion work $W_e = Z W_c$; total fluctuation of speed ±4%.

- **To Find:** Moment of inertia of flywheel I.

Formula:

$$W_{net} = 120P/N \ \text{J/cycle}$$
$$W_c = W_{net}/(Z - 1), \ W_e = Z W_c$$

$\Delta E = W_e - (W_e - W_c)/4 = (3W_e + W_c)/4$

$C_s = 2(4)/100 = 0.08$

$I = \Delta E / (\omega^2 C_s)$

Solution:

1. For a four-stroke engine, cycles per minute = N/2:

   $P = W_net\,(N/2)/60 \Rightarrow W_net = 120P/N$

2. Since $W_e = Z\,W_c$ and $W_net = W_e - W_c$:

   $W_c = W_net/(Z - 1)$

3. Calculate maximum fluctuation of energy from TMD:

   $\Delta E = (3W_e + W_c)/4$

4. For ±4% fluctuation:

   $C_s = 0.08$

5. Calculate I:

   $I = \Delta E / (\omega^2 C_s),\ \omega = 2\pi N/60$

Final Answer:

$I = \Delta E / (\omega^2 (0.08))$

---

3. Turning Moment Diagram – Single-cylinder vs Multi-cylinder:

- **Q.** Compare the characteristics of turning moment diagram for a single cylinder engine and multicylinder engine. **(SUMMER 2026, 4 Marks)**

Answer:

Single-cylinder engine	Multi-cylinder engine
One power stroke per two revolutions	Power strokes overlap
Large torque fluctuation	More uniform torque
Large flywheel required	Smaller flywheel required
Higher speed fluctuation	Smoother operation

---

4. Flywheel in Punching Presses (Theory):

- **Q.** Analyze the role of a flywheel in punching presses. **(SUMMER 2026, 4 Marks OR)**

Answer:

- **Punching operation:** Energy required only during short punching interval; remaining cycle is idle.
- **Energy storage:** Flywheel stores energy during idle period from the motor.
- **Energy supply:** During punching, flywheel supplies additional energy rapidly.
- **Result:** Motor size and power rating are reduced; speed fluctuation is controlled and press runs smoothly.

Chapter 3: Balancing of Inertia Forces and Couples

Repeated Questions:

1. Balancing of Masses in Different Planes (Numerical):

- **Q.** X number of masses (with given mass, radius, angular position, and axial plane distances) are to be completely balanced by two masses in planes P and Q. Find the magnitude and angular position of the balancing masses. **(WINTER 2022, 7 Marks; SUMMER 2023, 7 Marks; WINTER 2023, 7 Marks; WINTER 2025, 7 Marks)**

Answer:

Given:

Masses m_1, m_2, m_3 at radii r_1, r_2, r_3, angular positions θ_1, θ_2, θ_3, and axial distances l_1, l_2, l_3 from reference plane. Balancing masses in planes P and Q at radii r_P, r_Q and distances l_P, l_Q.

- **To Find:** Magnitude and angular position of balancing masses m_P, m_Q.

Formulae:

$$\text{Centrifugal force } F_i = m_i\, r_i\, \omega^2$$

$$\text{Couple } C_i = F_i\, l_i$$

For complete balance:

$$\Sigma F = 0,\ \Sigma M = 0$$

Solution:

1. Draw the plane diagram and angular position diagram.
2. Tabulate $m_i r_i$, l_i, and $m_i r_i l_i$.
3. Resolve $m_i r_i$ and $m_i r_i l_i$ into horizontal and vertical components.
4. Take moment about plane P to find balancing mass in plane Q:

$$m_Q\, r_Q = \sqrt{[(\Sigma m_i r_i l_i)_H^2 + (\Sigma m_i r_i l_i)_V^2]} \,/\, l_Q$$

$$\tan\theta_Q = (\Sigma m_i r_i l_i)_V \,/\, (\Sigma m_i r_i l_i)_H$$

5. Take moment about plane Q to find balancing mass in plane P.
6. Check by force polygon.

Final Answer:

$$m_P = \sqrt{[(\Sigma m_i r_i l_iQ)_H^2 + (\Sigma m_i r_i l_iQ)_V^2]} \,/\, (r_P\, l_P)$$

$$m_Q = \sqrt{[(\Sigma m_i r_i l_iP)_H^2 + (\Sigma m_i r_i l_iP)_V^2]} \,/\, (r_Q\, l_Q)$$

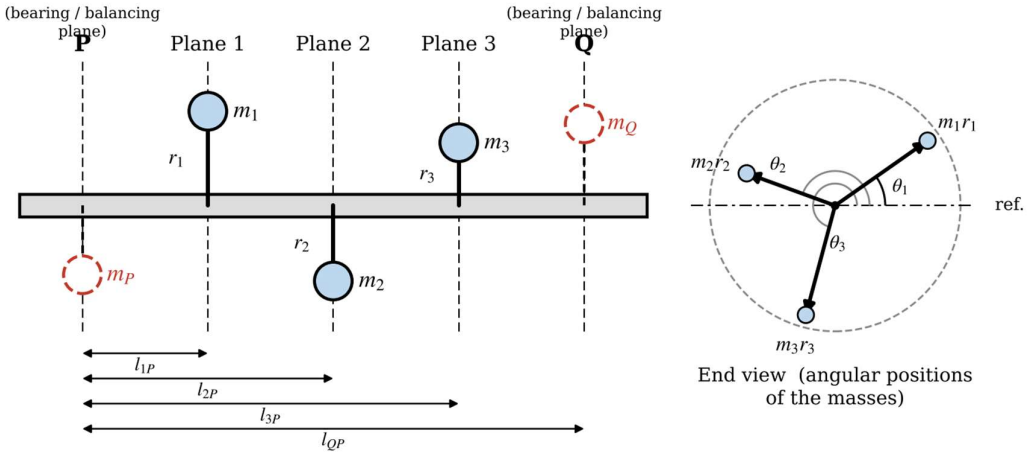


Fig. 6: Balancing of several masses in different planes (side view and end view)

2. Static and Dynamic Balancing:

- **Q.** Difference between Static and Dynamic Balancing. **(SUMMER 2022, 4 Marks; SUMMER 2024, 4 Marks; WINTER 2025, 3 Marks; SUMMER 2026, 3 Marks)**

Answer:

Static Balancing	Dynamic Balancing
Resultant centrifugal force is zero	Resultant force and resultant couple both zero
Σmr = 0	Σmr = 0 and Σmrl = 0
For rotating masses in same plane or thin discs	For long rotors with masses in different planes
Only force balance	Force and moment balance

3. Partial Balancing of Reciprocating Masses:

- **Q.** Explain why only a part of the unbalanced force due to reciprocating masses is balanced. **(SUMMER 2022, 4 Marks; WINTER 2022, 3 Marks)**

Answer:

Primary unbalanced force due to reciprocating mass varies in magnitude and direction:

$$F_p = m_{rec}\,\omega^2\,r\cos\theta$$

If fully balanced by a revolving mass, the counterbalance force also has a component perpendicular to the line of stroke, causing unbalanced force in the transverse direction.

Therefore, only a fraction c (usually c = 2/3) of reciprocating mass is balanced to reduce primary force without introducing excessive transverse force.

The unbalanced primary force remains:

$$(1 - c)\,m_{rec}\,\omega^2\,r\cos\theta$$

The balancing mass also produces a transverse force:

$$c\,m_{rec}\,\omega^2\,r\sin\theta$$

4. Balancing of Inline and V-Engines:

- **Q.** Explain the procedure of balancing of inline engines / V-engines. **(SUMMER 2022, 7 Marks; SUMMER 2023, 4 Marks; WINTER 2023, 4 Marks; SUMMER 2025, 7 Marks)**

Answer:

Inline engines:

- For multi-cylinder inline engine, arrange cranks symmetrically.
- **Primary force balance:** Sum of primary forces must be zero:

  $$\Sigma\,m_{rec}\,\omega^2\,r\cos\theta = 0$$
- **Primary couple balance:** Sum of moments of primary forces about any reference plane must be zero:

  $$\Sigma\,m_{rec}\,\omega^2\,r\,l\cos\theta = 0$$
- **Secondary force balance:** Sum of secondary forces at twice frequency must be zero:

  $$\Sigma\,(m_{rec}\,\omega^2\,r / n)\cos 2\theta = 0$$
- **Secondary couple balance:** Moments of secondary forces must balance.

V-engines:

- Two cylinders share a common crank at angle α between cylinder axes.
- Resolve inertia forces along the crank and perpendicular to it.
- **Primary force along crank:**

$F_p = m_{rec} \omega^2 r \cos\theta$

- For both cylinders, resultant primary force can be balanced by a counterweight on crank.
- Secondary forces act at twice crank speed and are not fully balanced by simple counterweight.

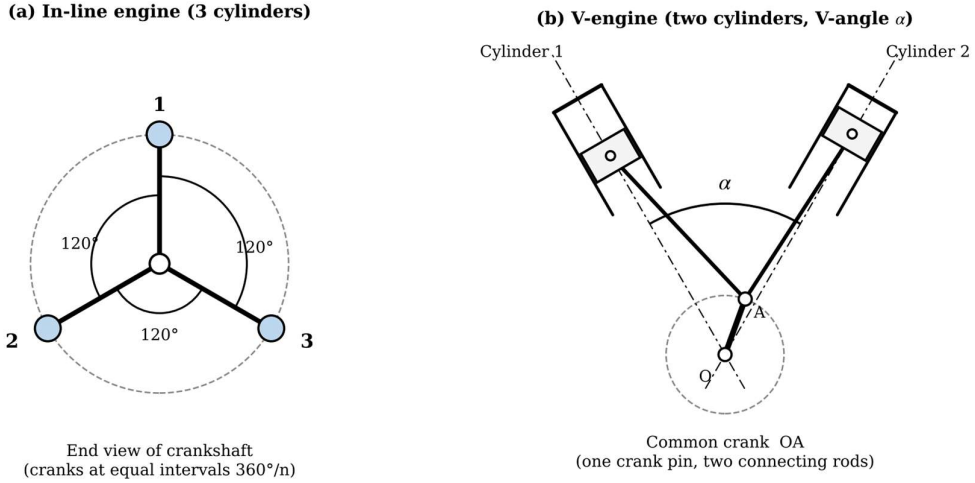


Fig. 7: Cranks of an in-line engine and cylinders of a V-engine

---

5. Balancing of Multi-cylinder Inline Engine (Primary/Secondary Forces):

- **Q.** For an in-line four/six-cylinder engine, examine the engine for the balance of primary and secondary forces and couples. Find the maximum unbalanced forces and couples. **(WINTER 2023, 7 Marks; WINTER 2024, 7 Marks)**

Answer:

- **Primary force balance condition:**
  $\Sigma m_i r_i \cos\theta_i = 0, \ \Sigma m_i r_i \sin\theta_i = 0$
- **Primary couple balance condition:**
  $\Sigma m_i r_i l_i \cos\theta_i = 0, \ \Sigma m_i r_i l_i \sin\theta_i = 0$
- **Secondary force balance condition:**
  $\Sigma (m_i r_i/n) \cos 2\theta_i = 0, \ \Sigma (m_i r_i/n) \sin 2\theta_i = 0$
- **Secondary couple balance condition:**
  $\Sigma (m_i r_i l_i/n) \cos 2\theta_i = 0, \ \Sigma (m_i r_i l_i/n) \sin 2\theta_i = 0$

Four-cylinder inline engine:

- Cranks at 0°, 180°, 180°, 0°.
- **Primary forces:** Balanced.
- **Primary couples:** Balanced.
- **Secondary forces:** Unbalanced (all act in same direction at twice frequency).
- **Secondary couples:** Balanced for symmetrical firing order.

Six-cylinder inline engine:

- Cranks at 0°, 120°, 240°, 240°, 120°, 0°.
- Primary and secondary forces and couples are balanced.
- **Final Result:** For four-cylinder engine, maximum unbalanced secondary force:
  $F_{s,max} = 4 \, m_{rec} \omega^2 r / n$

---

6. Need and Effects of Balancing:

- **Q.** Explain the need for balancing and the effects of unbalancing. **(SUMMER 2022, 3 Marks; WINTER 2023, 3 Marks)**

Answer:

Need for balancing:

- To reduce vibration and noise.
- To reduce wear of bearings and shafts.
- To improve fatigue life and comfort.

Effects of unbalancing:

- Excessive vibration and noise.
- Bearing loads and stresses increase.
- Loosening of fasteners, fatigue failure.
- Discomfort to passengers and damage to surroundings.

Other Important Questions:

1. Balancing of Rotating Parts in High-Speed Systems:

- **Q.** "The balancing of rotating parts is crucial for high speed systems." Justify the statement. **(WINTER 2025, 4 Marks)**

Answer:

Centrifugal force due to unbalance:

$$F = mr\omega^2$$

- It increases with square of speed.
- At high speed, even a small eccentricity produces very large force.
- This causes severe vibration, bearing wear, and possible failure.
- Hence dynamic balancing is essential for high-speed rotors, turbines, and engines.

---

2. Balancing of Reciprocating Masses (Theory):

- **Q.** Explain the concept of primary and secondary balancing in reciprocating engines. **(SUMMER 2026, 4 Marks)**

Answer:

Primary balancing:

Due to reciprocating mass, primary force:

$$F_p = m_rec\ \omega^2\ r\ \cos\theta$$

Balanced partially or fully by counterweight on crank.

Secondary balancing:

Due to obliquity of connecting rod, secondary force:

$$F_s = (m_rec\ \omega^2\ r\ /\ n)\ \cos 2\theta$$

- Frequency is twice the crank speed.
- Balanced using an imaginary crank rotating at 2ω with radius r/(4n).

- **Q.** Explain the partial balancing of primary unbalanced forces in single cylinder reciprocating engine. **(SUMMER 2026, 4 Marks OR)**

Answer:

Let fraction c of reciprocating mass be balanced.

Balancing mass m_b at radius r_b:

$$c\, m_rec\, r = m_b\, r_b$$

Unbalanced primary force along line of stroke:

$$F_unbalanced = (1 - c)\, m_rec\, \omega^2\, r \cos\theta$$

Balancing mass produces transverse force:

$$F_transverse = c\, m_rec\, \omega^2\, r \sin\theta$$

Resultant unbalanced force:

$$F_R = m_rec\, \omega^2\, r \sqrt{[(1 - c)^2 \cos^2\theta + c^2 \sin^2\theta]}$$

- **Q.** Define imaginary crank using neat sketch with regard to the balancing of reciprocating secondary unbalanced forces. **(SUMMER 2026, 3 Marks OR)**

Answer:

- **Imaginary crank:** A fictitious crank rotating at twice the engine speed 2ω, used to represent the secondary unbalanced force.

Radius of imaginary crank:

$$r_i = r/(4n)$$

Secondary force:

$$F_s = m_rec\, (2\omega)^2\, r_i \cos 2\theta = (m_rec\, \omega^2\, r / n) \cos 2\theta$$

It rotates at 2ω and is used to balance secondary forces in multi-cylinder engines.

---

3. Partial Balancing in Locomotives (Theory):

- **Q.** Explain the effects of partial balancing in locomotives (Variation of Tractive Force, Swaying Couple). **(WINTER 2024, 4 Marks)**

Answer:

- **Variation of tractive force:** Unbalanced primary force causes fluctuation in tractive effort.
- **Swaying couple:** Unbalanced forces on opposite sides create a couple tending to sway the locomotive.
- **Hammer blow:** Unbalanced vertical force causes variation in load on rails, leading to hammering action.
- These effects cause track damage and uncomfortable riding.

- **Q.** Derive the equation for the following terms pertaining to locomotives: i) Tractive force ii) Swaying couple iii) Hammer blow. **(WINTER 2025, 7 Marks OR)**

Answer:

Given:

Two cylinders, crank radius r, distance between cylinder centres a, fraction c of reciprocating mass balanced.

1. Tractive force:

$$F_T = F_L + F_R$$

$$F_T = (1 - c)\, m_rec\, \omega^2\, r\, (\cos\theta + \cos(90° + \theta))$$

For 90° crank angle:

$$F_T = (1 - c)\, m_rec\, \omega^2\, r\, (\cos\theta - \sin\theta)$$

2. Swaying couple:

Moments about centre:

$$M = F_L\, (a/2) - F_R\, (a/2)$$

$$M = (1 - c)\, m_rec\, \omega^2\, r\, (a/2)\, (\cos\theta + \sin\theta)$$

3. Hammer blow:

Vertical unbalanced force due to balance mass:

$$F_b = m_b \omega^2 r_b \sin\theta$$

Maximum hammer blow:

$$F_{b,max} = c\, m_{rec} \omega^2 r$$

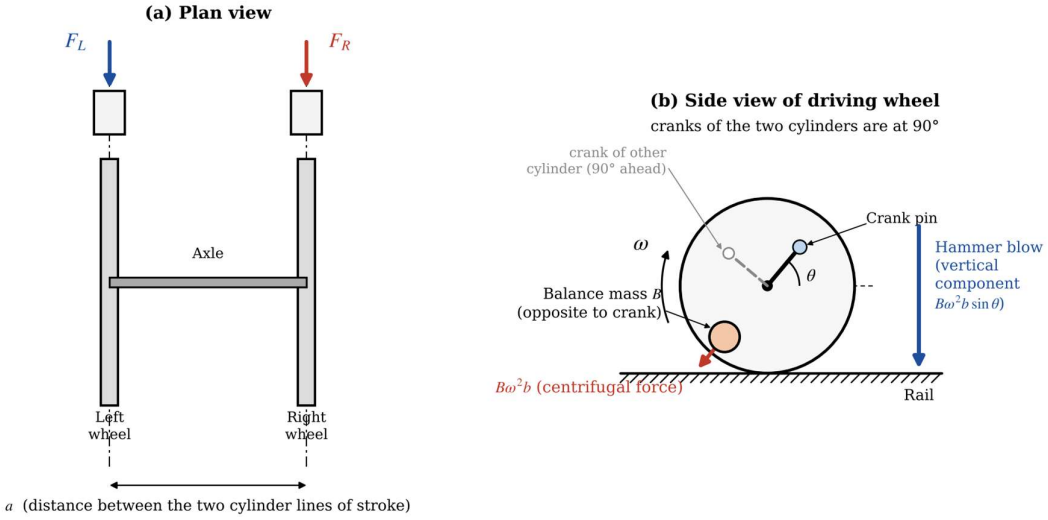


Fig. 8: Hammer blow in a two-cylinder locomotive

---

4. Locomotive Balancing (Numerical):

- **Q.** For an inside cylinder locomotive, if X% of reciprocating masses are to be balanced, determine the magnitude and angular position of balanced masses in the plane of wheels. **(SUMMER 2025, 7 Marks)**

Answer:

[Answer not found in the answer chat – to be added]

- **Q.** The following data refer to an outside cylinder uncoupled locomotive: Mass of rotating parts per cylinder = 350 kg, Mass of reciprocating parts per cylinder = 300 kg, Angle between cranks = 90°, Crank radius = 0.3 m, Cylinder centers = 1.8 m, Radius of balance masses = 0.8 m, Wheel centers = 1.5 m. If whole of the rotating and 2/3rd of the reciprocating parts is to be balanced in planes of the driving wheels, find (a) magnitude and angular positions of balance masses, (b) speed in km/h at which the wheel will lift off the rails when the load on each driving wheel is 30 kN, and the diameter of tread of driving wheels is 1.8 m. **(SUMMER 2026, 7 Marks)**

Answer:

Given:

- Mass of rotating parts per cylinder, m_rot = 350 kg
- Mass of reciprocating parts per cylinder, m_rec = 300 kg
- Angle between cranks = 90°
- Crank radius, r = 0.3 m
- Cylinder centres = 1.8 m
- Radius of balance masses, r_b = 0.8 m
- Wheel centres = 1.5 m
- Whole rotating and 2/3 reciprocating parts to be balanced.
- Load on each driving wheel = 30 kN, tread diameter = 1.8 m.

To Find:

(a) Magnitude and angular positions of balance masses

(b) Speed at which wheel lifts off rails

Solution:

Balanced reciprocating mass per cylinder:

$$m_{bal,rec} = (2/3)(300) = 200 \text{ kg}$$

Total mass to be balanced per cylinder:

$$m = m_{rot} + m_{bal,rec} = 350 + 200 = 550 \text{ kg}$$

For two cylinders at 90°, resultant balancing mass in each wheel plane:

$$m_b r_b = \sqrt{[(550r)^2 + (550r)^2]} = 550r\sqrt{2}$$

$$m_b = 550(0.3)\sqrt{2} / 0.8 = 291.6 \text{ kg}$$

Angular position: each balance mass is opposite to the resultant of the two crank masses; since cranks are at 90°, resultant is at 45° from each crank.

Speed for wheel lift: Hammer blow equals load on wheel:

$$c\, m_{rec}\, \omega^2\, r = 30 \times 10^3$$

$$\omega = \sqrt{[30000 / ((2/3)(300)(0.3))]} = 22.36 \text{ rad/s}$$

$$N = 60\omega/2\pi = 213.5 \text{ rpm}$$

$$v = \pi DN/60 = \pi(1.8)(213.5)/60 = 20.1 \text{ m/s} = 72.4 \text{ km/h}$$

Final Answer:

$m_b = 291.6$ kg at 45° to each crank

$v = 72.4$ km/h

---

5. V-Engine Balancing (Numerical):

- **Q.** A twin cylinder V-engine has the cylinders set at an angle of 45°. The connecting rods are 30 cm long and the crank radius is 70 mm. The total rotating mass is equivalent to 3 kg at the crank radius and the reciprocating parts mass is 2 kg per line. A balance mass is fitted opposite to the crank equivalent to 2.5 kg at a radius of 10 cm. The engine speed is 2000 rpm. What is the value of the maximum resultant secondary force? **(SUMMER 2026, 7 Marks OR)**

Answer:

Given:

- Angle between cylinders, $\alpha = 45°$
- Connecting rod length, $l = 30$ cm $= 0.3$ m
- Crank radius, $r = 70$ mm $= 0.07$ m
- Rotating mass equivalent at crank radius, $m_{rot} = 3$ kg
- Reciprocating mass per line, $m_{rec} = 2$ kg
- Balance mass, $m_b = 2.5$ kg at $r_b = 10$ cm $= 0.1$ m
- Speed, $N = 2000$ rpm
- **To Find:** Maximum resultant secondary force.

Formula:

$$n = l/r = 0.3/0.07 = 4.286$$

$$\omega = 2\pi(2000)/60 = 209.44 \text{ rad/s}$$

Secondary force for each cylinder:

$$F_{s1} = (m_{rec}\, \omega^2\, r / n) \cos 2\theta$$

$$F_{s2} = (m_{rec}\, \omega^2\, r / n) \cos 2(\theta + \alpha)$$

Resultant secondary force:

$$F_s = \sqrt{(F_{s1}^2 + F_{s2}^2 + 2\, F_{s1}\, F_{s2} \cos\alpha)}$$

Solution:

Calculate secondary force amplitude:

$$F_{s0} = 2(209.44)^2(0.07)/4.286 = 1433.6 \text{ N}$$

For maximum resultant, take $\theta = 0°$:

$$F_{s1} = 1433.6 \cos 0° = 1433.6 \text{ N}$$

$$F_{s2} = 1433.6 \cos(2 \times 45°) = 1433.6 \cos 90° = 0$$

Resultant:

$$F_s = \sqrt{(1433.6^2 + 0 + 0)} = 1433.6 \text{ N}$$

Check other positions: maximum occurs when $\cos 2\theta = 1$ and $\cos 2(\theta + \alpha) = 1$, but for $\alpha = 45°$, $\cos 90° = 0$. Hence maximum is 1433.6 N.

Final Answer:

$$F_{s,max} = 1433.6 \text{ N} \approx 1.43 \text{ kN}$$

Chapter 4: Gyroscope

Repeated Questions:

1. Gyroscopic Effect on a Naval Ship (Numerical):

- **Q.** The rotor of a turbine of a ship has a mass M, radius of gyration k, and speed N rpm. Determine the gyroscopic couple and its effect when: (a) The ship steers left/right on a curve of radius R at speed V. (b) The ship pitches with a simple harmonic motion (given period and angular displacement). (c) The ship rolls. **(WINTER 2022, 7 Marks; SUMMER 2024, 7 Marks; WINTER 2024, 7 Marks; SUMMER 2025, 7 Marks; WINTER 2025, 7 Marks)**

Answer:

Given:

Rotor mass m, radius of gyration k, speed N rpm

$$I = mk^2,\ \omega = 2\pi N/60$$

Ship speed V, curve radius R, pitching period t_p, angular displacement θ, rolling amplitude ϕ.

- **To Find:** Gyroscopic couple and its effect for (a) steering, (b) pitching, (c) rolling.

Formulae:

$$C = I\ \omega\ \omega_p$$

(a) Steering: Precession angular velocity

$$\omega_p = V/R$$

$$C_s = I\ \omega\ V/R$$

- **Effect:** Nose tends to dip or rise depending on direction of turn and rotor rotation.

(b) Pitching: Simple harmonic pitching:

$$\omega_p = (2\pi/t_p)\ \theta_{max}$$

$$C_p = I\ \omega\ \omega_p$$

- **Effect:** Ship tends to turn towards port or starboard.

(c) Rolling:

$$\omega_r = (2\pi/t_r)\ \varphi_{max}$$

$$C_r = I\ \omega\ \omega_r$$

- **Effect:** Ship tends to pitch at bow or stern.

Solution:

1. Calculate $I = mk^2$.
2. Calculate spin angular velocity $\omega = 2\pi N/60$.
3. For steering: $\omega_p = V/R$, then $C = I\ \omega\ \omega_p$.
4. For pitching: $\omega_p = (2\pi/t_p)\theta$, then $C = I\ \omega\ \omega_p$.
5. For rolling: $\omega_r = (2\pi/t_r)\phi$, then $C = I\ \omega\ \omega_r$.
6. Determine effect direction using right-hand rule: spin vector, precession vector, couple vector.

Final Answer:

$$C_s = I\ \omega\ V/R,\ C_p = I\ \omega\ 2\pi\theta/t_p,\ C_r = I\ \omega\ 2\pi\varphi/t_r$$

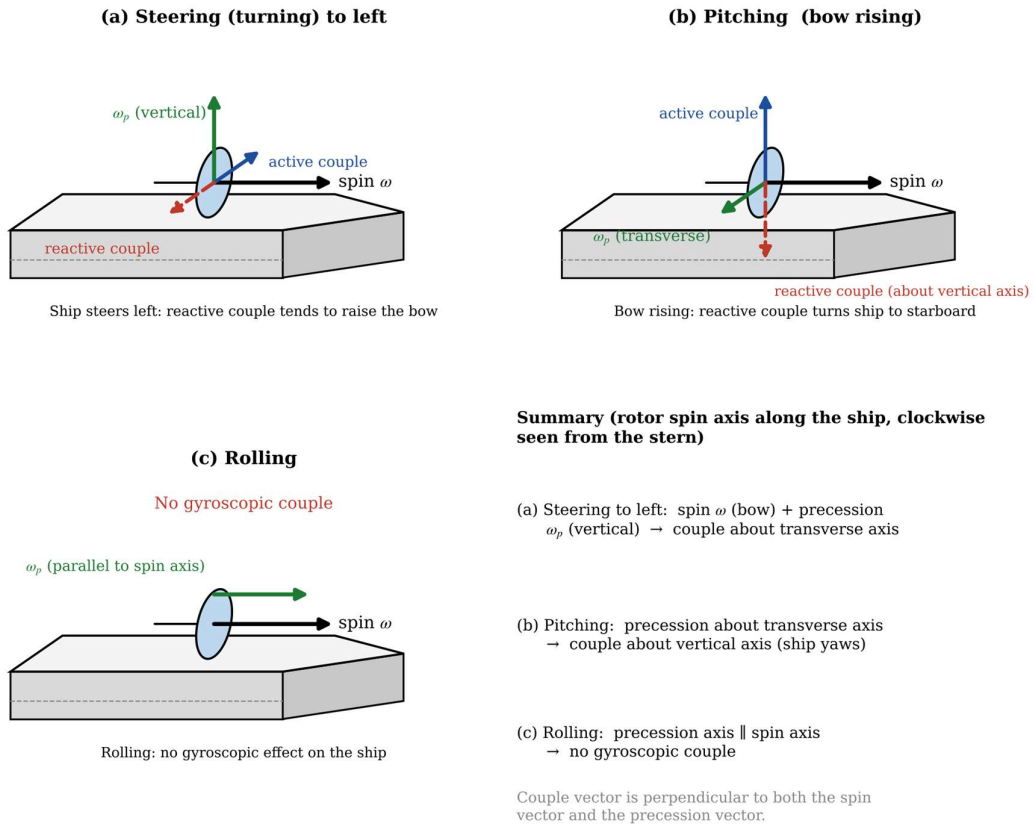


Fig. 9: Gyroscopic effect on a ship: steering, pitching and rolling

---

2. Gyroscopic Couple:

- **Q.** What do you understand by a gyroscopic couple? Derive a formula for its magnitude. **(SUMMER 2022, 4 Marks; SUMMER 2023, 4 Marks)**

Answer:

- **Gyroscopic Couple:** The couple produced on a rotating body when its axis of spin is forced to precess.

Derivation:

1. Consider a rotor rotating at angular velocity ω about spin axis.
2. Let the axis precess with angular velocity ω_p.
3. Angular momentum H = Iω.
4. Due to precession, change in angular momentum in time dt:

   $dH = H\, d\theta = H\, \omega_p\, dt$
5. Rate of change of angular momentum:

   $C = dH/dt = H\, \omega_p$

Therefore:

$C = I\, \omega\, \omega_p$

Final Answer:

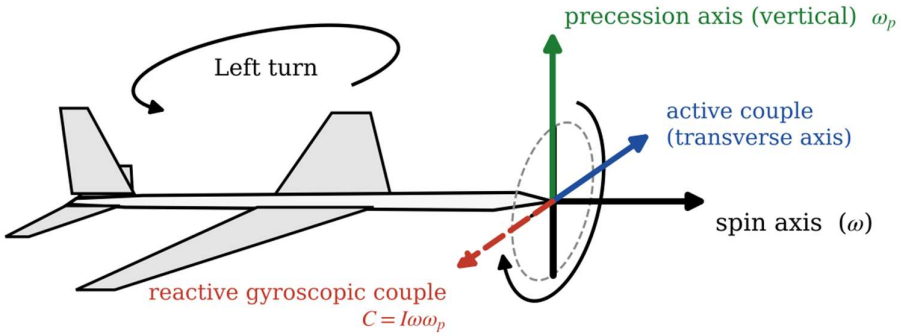
$C = I\, \omega\, \omega_p$

---

3. Gyroscopic Effect on an Aeroplane:

- **Q.** Explain the effect of the gyroscopic couple on an aircraft while taking a turn. (Often requires a sketch and stating the equation.) **(SUMMER 2022, 4 Marks; SUMMER 2025, 4 Marks)**

Answer:



Aeroplane taking a left turn (propeller rotating clockwise as seen by the pilot): the reactive couple acts about the transverse axis and tends to raise the nose.

Fig. 10: Gyroscopic effect on an aeroplane taking a left turn

Explanation:

- Aeroplane turns left/right; rotor spin axis precesses.
- **Gyroscopic couple:**
  C = I ω ω_p, ω_p = V/R
- For left turn with clockwise propeller (viewed from rear), nose tends to rise or dip depending on direction.
- Effect causes tendency to pitch nose up or down, which must be balanced by control surfaces.
- Same principle applies to pitch and yaw motions of aircraft.

---

4. Active and Reactive Gyroscopic Couple:

- **Q.** Explain with a neat sketch: (i) Active Gyroscopic Couple (ii) Reactive Gyroscopic Couple. **(WINTER 2022, 4 Marks; WINTER 2023, 4 Marks)**

Answer:

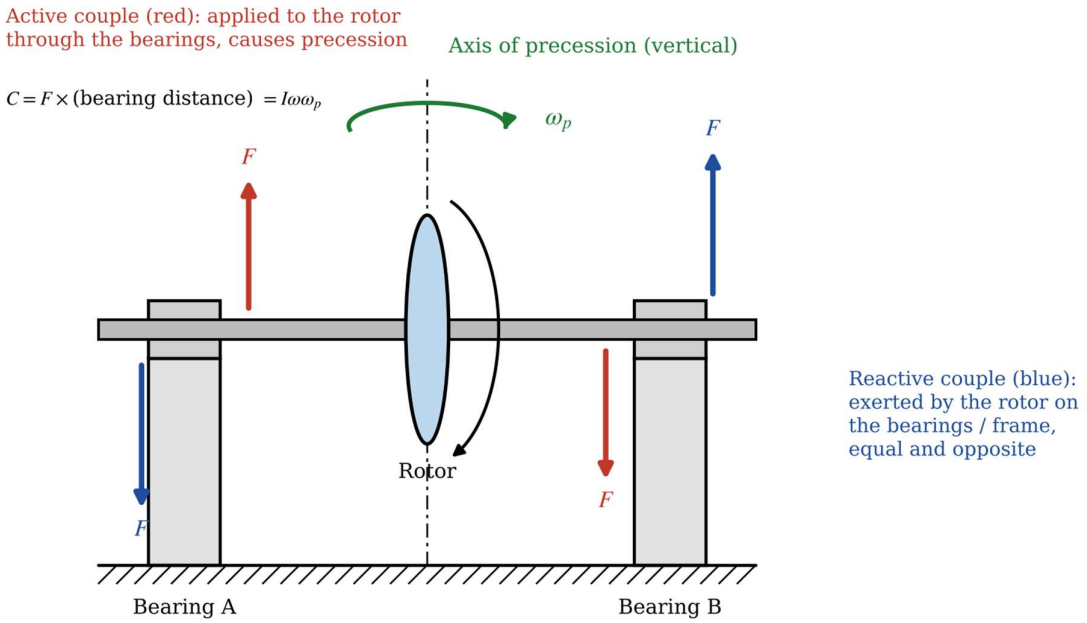


Fig. 11: Active and reactive gyroscopic couple on a rotor mounted in bearings

Active Gyroscopic Couple	Reactive Gyroscopic Couple
Couple applied to precess the spin axis	Couple exerted by rotor on bearings
Produced by external agency	Equal and opposite reaction
Causes precession	Tends to rotate the support

Explanation:

- **Active couple:** External couple required to rotate the spin axis.
- **Reactive couple:** Rotor exerts equal and opposite couple on bearings/support.
- Both have magnitude C = I ω ω_p.

Other Important Questions:

1. Gyroscopic Effect on an Automobile / Four-Wheel Vehicle:

- **Q.** Explain the effect of the gyroscopic and centrifugal couple on the reaction of the four wheels of a vehicle negotiating a curve. **(SUMMER 2022, 7 Marks)**

Answer:

Given:

Vehicle mass m, wheel track x, CG height h, speed V, curve radius R, moment of inertia of rotating parts I, engine speed ω, wheel radius r.

- **To Find:** Load on each wheel.

Formulae:

$\omega_p = V/R$

Gyroscopic couple:

$$C_g = I \omega \omega_p$$

Centrifugal force:

$$F_c = mV^2/R$$

Overturning couple due to centrifugal force about outer wheel:

$$C_c = F_c h$$

Load transfer due to gyroscopic couple and centrifugal force.

Solution:

1. Calculate gyroscopic couple $C_g = I \omega \omega_p$.
2. Determine direction of gyroscopic couple: if engine rotates clockwise viewed from front and vehicle turns left, gyroscopic couple tends to lift inner or outer wheels.
3. Calculate centrifugal force F_c.
4. Load on outer wheels:

$$W_{outer} = W/2 + (C_c \pm C_g)/x$$

5. Load on inner wheels:

$$W_{inner} = W/2 - (C_c \pm C_g)/x$$

Sign depends on direction of turn and rotor rotation.

Final Answer:

$$W_{outer} = mg/2 + (F_c h \pm C_g)/x, \quad W_{inner} = mg/2 - (F_c h \pm C_g)/x$$

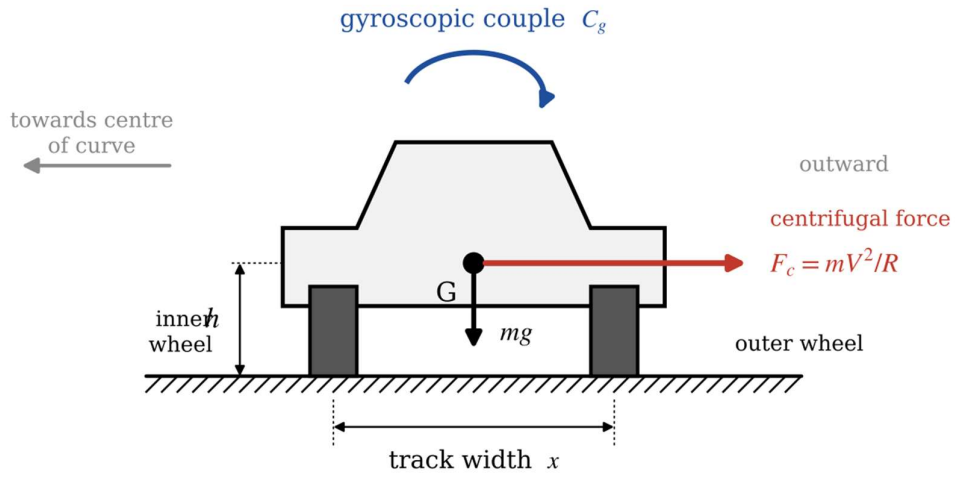


Fig. 12: Four-wheel vehicle negotiating a curve (front view)

- **Q.** A four-wheeled motor car takes a turn. Find the load on each wheel. (Involves considering both centrifugal and gyroscopic effects.) **(SUMMER 2023, 7 Marks)**

Answer:

Given:

Vehicle mass m, wheel track x, CG height h, speed V, curve radius R, moment of inertia of rotating parts I, engine speed ω, wheel radius r.

- **To Find:** Load on each wheel.

Formulae:

$$\omega_p = V/R$$

Gyroscopic couple:

$C_g = I \omega \omega_p$

Centrifugal force:

$F_c = mV^2/R$

Overturning couple due to centrifugal force about outer wheel:

$C_c = F_c h$

Load transfer due to gyroscopic couple and centrifugal force.

Solution:

1. Calculate gyroscopic couple $C_g = I \omega \omega_p$.
2. Determine direction of gyroscopic couple: if engine rotates clockwise viewed from front and vehicle turns left, gyroscopic couple tends to lift inner or outer wheels.
3. Calculate centrifugal force F_c.
4. Load on outer wheels:

$W_{outer} = W/2 + (C_c \pm C_g)/x$

5. Load on inner wheels:

$W_{inner} = W/2 - (C_c \pm C_g)/x$

Sign depends on direction of turn and rotor rotation.

Final Answer:

$W_{outer} = mg/2 + (F_c h \pm C_g)/x, \; W_{inner} = mg/2 - (F_c h \pm C_g)/x$

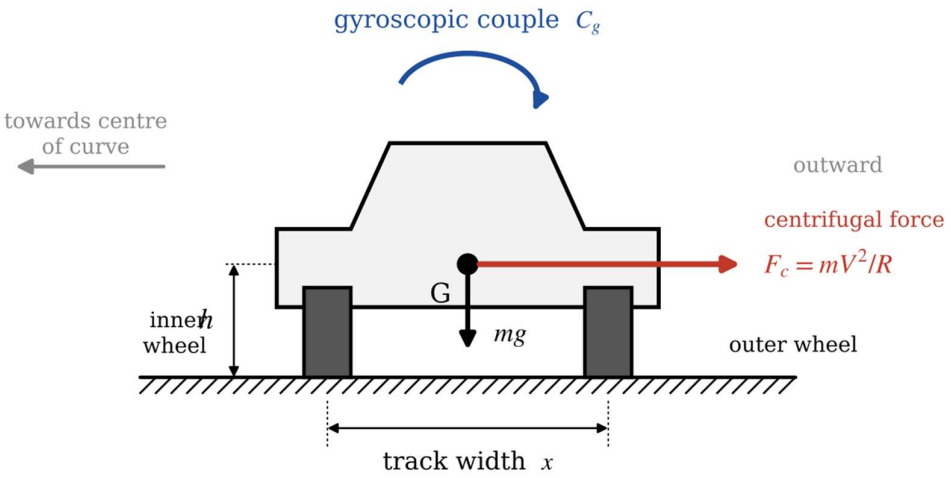


Fig. 13: Four-wheel vehicle negotiating a curve (front view)

---

2. Gyroscope Terminology:

- **Q.** Define: i) Axis of spin ii) Plane of Precession iii) Axis of Gyroscopic Reaction iv) Plane of Gyroscopic Couple. Show each in an illustrative diagram. **(WINTER 2025, 3 Marks)**

Answer:

- **Axis of spin:** Axis about which rotor rotates.
- **Plane of precession:** Plane in which axis of spin rotates due to external couple.
- **Axis of gyroscopic reaction:** Axis about which reactive gyroscopic couple acts.
- **Plane of gyroscopic couple:** Plane perpendicular to axis of gyroscopic reaction.

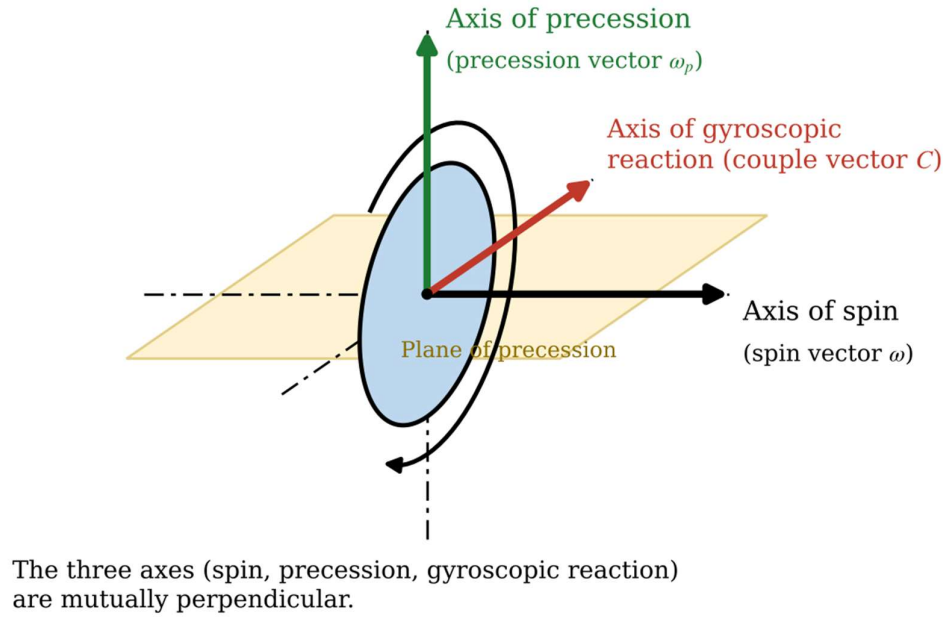


Fig. 14: Axis of spin, axis of precession and axis of gyroscopic reaction

---

3. Gyroscopic Effect on a Two-Wheeler:

- **Q.** Analyze the effect of gyroscopic couple on the stability of a two-wheeler during a turn. **(SUMMER 2026, 3 Marks)**

Answer:

- Two-wheeler wheels act as gyroscopes.
- **When taking a turn, gyroscopic couple:**
  $C = I \omega V/R$
- Couple tends to keep the vehicle upright or lean depending on direction.
- Rider leans inward; gyroscopic effect helps stabilize during turns.
- At high speed, gyroscopic effect is significant and improves stability.

---

4. Gyroscopic Effect on an Aeroplane (Numerical):

- **Q.** An aeroplane makes a complete half circle of 60 m radius towards left when flying at 250 km/h. The rotary engine and the propeller of the plane have a mass of 450 kg with a radius of gyration of 300 mm. The engine runs at 2400 rpm clockwise when viewed from the rear. Find the gyroscopic effect on the aircraft. **(SUMMER 2026, 4 Marks)**

Answer:

Given:

- Radius of turn R = 60 m (half circle)
- Speed V = 250 km/h = 69.44 m/s
- Mass of engine and propeller m = 450 kg
- Radius of gyration k = 300 mm = 0.3 m
- Engine speed N = 2400 rpm, clockwise viewed from rear.

- **To Find:** Gyroscopic effect on aircraft.

Formula:

$I = mk^2 = 450(0.3)^2 = 40.5\ kg{\cdot}m^2$

$\omega = 2\pi(2400)/60 = 251.33\ rad/s$

$\omega_p = V/R = 69.44/60 = 1.157\ rad/s$

$C = I\ \omega\ \omega_p$

Solution:

Calculate moment of inertia:

$I = 450(0.3)^2 = 40.5\ kg{\cdot}m^2$

Spin speed:

$\omega = 251.33\ rad/s$

Precession angular velocity for half circle turn:

$\omega_p = V/R = 69.44/60 = 1.157\ rad/s$

Gyroscopic couple:

$C = 40.5(251.33)(1.157) = 11778\ N{\cdot}m$

Direction: Engine rotates clockwise viewed from rear. Aircraft turns left. By right-hand rule, gyroscopic couple tends to raise the nose and dip the tail (or vice versa depending on convention). The effect is a pitching moment on the aircraft.

Final Answer:

$C = 11.78\ kN{\cdot}m$, causing a pitching tendency on the aircraft

---