GUJARAT TECHNOLOGICAL UNIVERSITY

IMP QUESTIONS – GTU BE SEMESTER 5 – 9 PAPERS (W22 TO S26) – ANSWER BANK

Subject Name & Code: Dynamics of Machinery - 3151911

2. Numerical types – ranked (study in this order)

1. Numerical Types Ranked by Frequency:

- **Q.** Numerical type – Damped free vibration (log decrement, damping factor, damped frequency, critical damping). Spring-mass with c given: find c_c, ωd, δ (W24, S26). 30 kg, 20 kN/m, ζ = 0.25 (W24, S26). Successive amplitudes given: find ζ and ωn (W23, S23, S25, S22, S24). Winter 2025 had damping as theory only. Learn: ωn = √(k/m), c_c = 2√(km), ζ = c/c_c, ωd = ωn√(1–ζ²), δ = ln(x1/x2) = 2πζ/√(1–ζ²). **(Rank 1; Papers: 8 of 9)**

Answer:

Given:

- Mass, m = 30 kg
- Spring stiffness, k = 20 kN/m = 20,000 N/m
- Damping factor, ζ = 0.25
- **To Find:** c_c, ω_n, ω_d, logarithmic decrement δ, successive amplitude ratio.

Formula:

$\omega_n = \sqrt{(k/m)},\ c_c = 2\sqrt{(km)} = 2m\omega_n$

$\omega_d = \omega_n \sqrt{(1 - \zeta^2)},\ \delta = 2\pi\zeta / \sqrt{(1 - \zeta^2)}$

Solution:

Natural frequency:

$\omega_n = \sqrt{(20000/30)} = 25.82\ \text{rad/s}$

$f_n = 25.82/2\pi = 4.11\ \text{Hz}$

Critical damping coefficient:

$c_c = 2\sqrt{(20000 \times 30)} = 2\sqrt{600000} = 2(774.6) = 1549.2\ \text{N·s/m}$

Actual damping coefficient:

$c = \zeta c_c = 0.25(1549.2) = 387.3\ \text{N·s/m}$

Damped frequency:

$\omega_d = 25.82\sqrt{(1 - 0.25^2)} = 25.82\sqrt{0.9375} = 25.82(0.9682) = 25.00\ \text{rad/s}$

$f_d = 25.00/2\pi = 3.98\ \text{Hz}$

Logarithmic decrement:

$\delta = 2\pi(0.25)/\sqrt{(1 - 0.25^2)} = 1.5708/0.9682 = 1.622$

Successive amplitude ratio:

$x_1/x_2 = e^{\delta} = e^{1.622} = 5.06$

Final Answer:

$\omega_n = 25.82\ \text{rad/s},\ c_c = 1549.2\ \text{N·s/m}$

$\omega_d = 25.00\ \text{rad/s},\ \delta = 1.622,\ x_1/x_2 = 5.06$

- **Q.** Numerical type – Gyroscopic effect. Ship turbine rotor: steering, pitching, rolling (W25, W24, W23, W22, S25, S24). Aeroplane half circle / turn (W25, S26). Four-wheeler on curve: load on

each wheel / limiting speed (S23, S24). Learn: C = Iωωp, I = mk², ωp for steering = v/R, pitching ωp max (SHM), direction rules. **(Rank 2; Papers: 8 of 9)**

Answer:

Given:

- Mass moment of inertia of rotor, I = 4000 kg·m²
- Speed of rotor, N = 3000 rpm
- Ship speed, v = 36 km/h = 10 m/s
- Radius of turn, R = 200 m
- **To Find:** Gyroscopic couple during steering, C.

Formula:

$\omega = 2\pi N/60,\ \omega_p = v/R,\ C = I\omega\omega_p$

Solution:

Spin angular velocity:

$\omega = 2\pi(3000)/60 = 314.16\ \text{rad/s}$

Precession angular velocity:

$\omega_p = 10/200 = 0.05\ \text{rad/s}$

Gyroscopic couple:

$C = (4000)(314.16)(0.05)$

$C = 62{,}832\ \text{N·m}$

Convert to kN·m:

$C = 62.83\ \text{kN·m}$

Final Answer:

$C \approx 62.8\ \text{kN·m}$

Diagram Description: A ship with a turbine rotor spinning about the longitudinal axis. During steering, the rotor axis precesses about a vertical axis with ω_p = v/R; the gyroscopic couple acts in a vertical plane and tends to raise or dip the bow.

Gyroscope Formula Bank

- **Mass moment of inertia:** I = mk²
- **Spin angular velocity:** ω = 2πN/60
- **Precession during steering:** ω_p = v/R
- **Precession during pitching:** ω_p = ω_p,max sin ωt
- **Gyroscopic couple:** C = Iωω_p
- **For ship steering:** C = Iω(v/R)
- **For aeroplane turn:** C = Iω(v/R)
- **For vehicle on curve:** C = Iω(v/R)

- **Q.** Numerical type – Critical speed of shaft (Dunkerley, Rayleigh, whirling, eccentric rotor). Three discs, Dunkerley (W25, S25). Two discs/wheels with shaft mass (S24, S26). Eccentric rotor at mid-span: amplitude and bearing force (W24 ×2, W23). Cantilever shaft with disc: longitudinal + transverse frequency (W25, S24). **(Rank 3; Papers: 7 of 9)**

Answer:

Given:

- Shaft diameter, d = 50 mm = 0.05 m
- Length, L = 1.0 m
- Young's modulus, E = 200 GPa = 200 × 10^9 N/m²

- Rotor mass, m = 100 kg at mid-span
- **To Find:** Critical speed N_c.

Formula:

$I = \pi d^4/64,\ \delta = mgL^3/(48EI),\ \omega_c = \sqrt{(g/\delta)},\ N_c = 60\omega_c/2\pi$

Solution:

Moment of inertia:

$I = \pi(0.05)^4/64 = \pi(6.25 \times 10^{-6})/64 = 3.068 \times 10^{-7}\ m^4$

Static deflection:

$\delta = 100(9.81)(1.0)^3 / (48(200 \times 10^9)(3.068 \times 10^{-7}))$

$\delta = 981 / (48(200 \times 10^9)(3.068 \times 10^{-7}))$

$\delta = 981 / 2945280 = 3.331 \times 10^{-4}\ m$

Critical angular speed:

$\omega_c = \sqrt{(9.81 / (3.331 \times 10^{-4}))} = \sqrt{29451} = 171.6\ rad/s$

Critical speed in rpm:

$N_c = 60(171.6)/2\pi = 1638.7\ rpm$

Final Answer:

$N_c \approx 1639\ rpm$

Diagram Description: A simply supported shaft of length 1.0 m with a 100 kg rotor at mid-span. The shaft deflects by 3.331 × 10⁻⁴ m under the rotor weight; at about 1639 rpm the shaft becomes dynamically unstable.

Given:

Simply supported shaft, length L = 1.2 m, diameter d = 40 mm = 0.04 m

E = 200 GPa

Disc 1: m_1 = 50 kg at a = 0.4 m from left bearing

Disc 2: m_2 = 80 kg at b = 0.8 m from left bearing

- **To Find:** Fundamental critical speed using Dunkerley's method.

Formula:

$I = \pi d^4/64,\ \delta_i = W_i x^2(L - x)^2 / (3EIL)$

$\omega_c = \sqrt{(g / (\delta_1 + \delta_2))}$

Solution:

Moment of inertia:

$I = \pi(0.04)^4/64 = 1.257 \times 10^{-7}\ m^4$

Deflection at disc 1 due to its own weight:

$\delta_1 = 50(9.81)(0.4)^2(1.2 - 0.4)^2 / (3(200 \times 10^9)(1.257 \times 10^{-7})(1.2))$

Numerator: 50(9.81)(0.16)(0.64) = 50(9.81)(0.1024) = 50.23

Denominator: 3(200 × 10⁹)(1.257 × 10⁻⁷)(1.2) = 3(200)(1.257)(1.2) × 10² = 3(301.68) × 10² = 90504

$\delta_1 = 50.23/90504 = 5.55 \times 10^{-4}\ m$

Deflection at disc 2 due to its own weight:

$\delta_2 = 80(9.81)(0.8)^2(1.2 - 0.8)^2 / (3(200 \times 10^9)(1.257 \times 10^{-7})(1.2))$

Numerator: 80(9.81)(0.64)(0.16) = 80(9.81)(0.1024) = 80.46

$\delta_2 = 80.46/90504 = 8.89 \times 10^{-4}\ m$

Sum of deflections:

$\delta_1 + \delta_2 = 5.55 \times 10^{-4} + 8.89 \times 10^{-4} = 1.444 \times 10^{-3}\ m$

Critical angular speed:

$\omega_c = \sqrt{(9.81 / (1.444 \times 10^{-3}))} = \sqrt{6793} = 82.42\ rad/s$

Critical speed in rpm:

$$N_c = 60(82.42)/2\pi = 787.3 \text{ rpm}$$

Final Answer:

$$N_c \approx 787 \text{ rpm}$$

Diagram Description: A simply supported shaft of length 1.2 m carrying a 50 kg disc at 0.4 m and an 80 kg disc at 0.8 m from the left bearing. Dunkerley's method sums the individual static deflections to estimate the fundamental critical speed.

Critical Speed Formula Bank

- **Single rotor at mid-span:**
  $\omega_c = \sqrt{(48EI/(mL^3))} = \sqrt{(g/\delta)}$
- **Cantilever shaft with end rotor:**
  $\omega_c = \sqrt{(3EI/(mL^3))}$
- **Dunkerley's method:**
  $1/\omega_c^2 = 1/\omega_1^2 + 1/\omega_2^2 + \cdots$
- **Rayleigh's method:**
  $\omega_c = \sqrt{(g \Sigma W_i y_i / \Sigma W_i y_i^2)}$
- **Eccentric rotor amplitude:**
  $y = e\omega^2 / (\omega_n^2 - \omega^2)$
- **Bearing force due to whirl:**
  $F = m\omega^2(y + e)$

- **Q.** Numerical type – Balancing of masses in different planes. Four masses A, B, C, D (200, 300, 400, 200 kg) in planes X and Y (W25, W22 – same). Four masses P1–P4 (W23). P, Q, R, S find P (S23). Pulleys / rotor in planes (W24 OR, S24 OR). Method: couple polygon first, then force polygon (tabular method). **(Rank 4; Papers: 6 of 9)**

Answer:

Given:

Masses and radii:

$m_A = 200$ kg, $r_A = 0.2$ m

$m_B = 300$ kg, $r_B = 0.15$ m

$m_C = 400$ kg, $r_C = 0.25$ m

$m_D = 200$ kg, $r_D = 0.3$ m

Angles measured from mass A:

$\theta_A = 0°$, $\theta_B = 90°$, $\theta_C = 180°$, $\theta_D = 270°$

Plane positions from A:

A = 0, B = 0.3 m, C = 0.6 m, D = 0.9 m

Balancing planes: X at 0.15 m, Y at 0.75 m from A.

Balancing radius $r_X = r_Y = 0.2$ m.

- **To Find:** Balancing masses m_X, m_Y and their angular positions.

Formula:

$$\Sigma mr = 0, \ \Sigma mrl = 0$$

Solution:

Force components mr:

$$A: 200(0.2) = 40 \text{ at } 0° \rightarrow (40, 0)$$

$$B: 300(0.15) = 45 \text{ at } 90° \rightarrow (0, 45)$$

C: $400(0.25) = 100$ at $180° \rightarrow (-100, 0)$

D: $200(0.3) = 60$ at $270° \rightarrow (0, -60)$

Sum of existing forces: (−60, −15).

Let X vector = (X_x, X_y), Y vector = (Y_x, Y_y).

$X_x + Y_x = 60,\ X_y + Y_y = 15 \quad (1)$

Couple components mrl:

A: 0

B: $45(0.3) = 13.5$ at $90° \rightarrow (0, 13.5)$

C: $100(0.6) = 60$ at $180° \rightarrow (-60, 0)$

D: $60(0.9) = 54$ at $270° \rightarrow (0, -54)$

Sum of existing couples: (−60, −40.5).

$0.15X_x + 0.75Y_x = 60,\ 0.15X_y + 0.75Y_y = 40.5 \quad (2)$

Solving (1) and (2):

$Y_x = 85,\ X_x = -25,\ Y_y = 63.75,\ X_y = -48.75$

Magnitudes:

$X = \sqrt{((-25)^2 + (-48.75)^2)} = 54.79$ kg·m

$Y = \sqrt{(85^2 + 63.75^2)} = 106.25$ kg·m

Balancing masses at r = 0.2 m:

$m_X = 54.79/0.2 = 273.95$ kg

$m_Y = 106.25/0.2 = 531.25$ kg

Angular positions:

$\theta_X = \tan^{-1}(-48.75/-25) = 242.8°$

$\theta_Y = \tan^{-1}(63.75/85) = 36.9°$

Final Answer:

$m_X \approx 274$ kg at $242.8°$, $m_Y \approx 531$ kg at $36.9°$

Diagram Description: Four masses A, B, C, D in parallel planes spaced along the shaft, with balancing masses in planes X and Y. The force polygon and couple polygon are drawn to close with the balancing vectors.

- **Q.** Numerical type – Flywheel. Turning moment areas +52, –124, +92, –140, +85, –72, +107: find flywheel mass (W25, S22 – same). Punching press 750 holes/h, 14000 Nm, 250/225 rpm: power and mass (W23, S25 – same). Engine flywheel mass / M.I. (W22 gas engine, S23 steam engine rim). **(Rank 5; Papers: 6 of 9)**

Answer:

Given:

Areas of turning moment diagram above/below mean torque:

+52, −124, +92, −140, +85, −72, +107 mm²

- **Scale:** 1 mm = 100 N·m (vertical), 1 mm = 1° (horizontal)
- Mean speed, N = 200 rpm
- Coefficient of fluctuation of speed, $C_s = 0.02$
- Radius of gyration, k = 0.5 m
- **To Find:** Mass of flywheel, m.

Formula:

$\Delta E = \text{max excess energy} - \text{min excess energy}$

$1\ \text{mm}^2 = 100 \times \pi/180 = 1.745$ J

$\Delta E = mk^2\omega^2 C_s,\ \omega = 2\pi N/60$

Solution:

Cumulative energy after each area:

$$0 \rightarrow +52 \rightarrow -72 \rightarrow +20 \rightarrow -120 \rightarrow -35 \rightarrow -107 \rightarrow 0$$

Maximum excess energy:

$$E_max = +52\ mm^2$$

Minimum excess energy:

$$E_min = -120\ mm^2$$

Fluctuation of energy:

$$\Delta E = (52 - (-120)) = 172\ mm^2$$

$$\Delta E = 172 \times 1.745 = 300.1\ J$$

Mean angular speed:

$$\omega = 2\pi(200)/60 = 20.944\ rad/s$$

Flywheel mass:

$$m = \Delta E / (k^2 \omega^2 C_s)$$

$$m = 300.1 / ((0.5)^2 (20.944)^2 (0.02))$$

$$m = 300.1 / (0.25 \times 438.65 \times 0.02)$$

$$m = 300.1 / 2.193 = 136.8\ kg$$

Final Answer:

$$m \approx 136.8\ kg \approx 137\ kg$$

Diagram Description: A turning moment diagram with mean torque line; positive areas +52, +92, +85, +107 and negative areas −124, −140, −72. Cumulative energy curve rises to a maximum after +52 and falls to a minimum after −140, giving ΔE = 172 mm².

Punching Press Numerical — Formula Bank

- **Energy required per hole:** E_hole = 14000 N·m
- **Number of holes per hour:** 750
- **Power of motor:**

  $$P = E_hole \times 750 / 3600\ W$$
- **Energy supplied by flywheel during punching:**

  $$\Delta E = E_hole - E_motor\ during\ punching$$
- **Flywheel mass:**

  $$\Delta E = m k^2 \omega^2 C_s$$
- **Speed fluctuation:**

  $$C_s = (N_1 - N_2)/N$$

- **Q.** Numerical type – Natural frequency of spring systems (series / parallel). K1–K6 with mass (W23); K1–K5, find m for 15 Hz (S22); pulley-spring (W22); lever with springs (S23); K1 K2 K3, 10 kg (S25); two springs with block (W24). Learn: equivalent stiffness for series and parallel, then ωn = √(keq/m). **(Rank 6; Papers: 6 of 9)**

Answer:

Given:

Springs $K_1 = 1000$ N/m, $K_2 = 2000$ N/m, $K_3 = 3000$ N/m

Mass, m = 10 kg

Case (a): All springs in series.

Case (b): K_1 and K_2 in parallel, then in series with K_3.

- **To Find:** Equivalent stiffness k_eq and natural frequency ω_n for each case.

Formula:

$1/k_eq = 1/K_1 + 1/K_2 + \cdots$ (series)

$k_eq = K_1 + K_2 + \cdots$ (parallel)

$\omega_n = \sqrt{(k_eq/m)}$

Solution:

Case (a): Series

$1/k_eq = 1/1000 + 1/2000 + 1/3000$

$1/k_eq = 0.001 + 0.0005 + 0.000333 = 0.001833$

$k_eq = 545.45\ N/m$

$\omega_n = \sqrt{(545.45/10)} = 7.39\ rad/s$

Case (b): Parallel $K_1 + K_2$, series with K_3

$K_{12} = 1000 + 2000 = 3000\ N/m$

$1/k_eq = 1/3000 + 1/3000 = 2/3000$

$k_eq = 1500\ N/m$

$\omega_n = \sqrt{(1500/10)} = 12.25\ rad/s$

Final Answer:

Case (a): $k_eq = 545.45\ N/m,\ \omega_n = 7.39\ rad/s$

Case (b): $k_eq = 1500\ N/m,\ \omega_n = 12.25\ rad/s$

- **Q.** Numerical type – Forced vibration – rotating unbalance / transmissibility. Unbalance 525 N at 6000 rpm, ζ = 0.3: amplitude and transmissibility (S25, S23 same type; W22 with 392 N, 3000 rpm). Compressor / engine on springs with damper: amplitude and phase (W23, S26). **(Rank 7; Papers: 5 of 9)**

Answer:

Given:

- Mass of system, m = 100 kg
- Spring stiffness, $k = 1 \times 10^6$ N/m
- Damping factor, ζ = 0.3
- Excitation force amplitude, $F_o = 525$ N
- Speed, N = 6000 rpm
- **To Find:** Amplitude X, transmissibility T_r, transmitted force F_T.

Formula:

$\omega = 2\pi N/60,\ \omega_n = \sqrt{(k/m)},\ r = \omega/\omega_n$

$X = F_0 / \sqrt{[(k - m\omega^2)^2 + (c\omega)^2]}$

$T_r = \sqrt{[(1 + (2\zeta r)^2) / ((1 - r^2)^2 + (2\zeta r)^2)]}$

Solution:

Excitation frequency:

$\omega = 2\pi(6000)/60 = 628.32\ rad/s$

Natural frequency:

$\omega_n = \sqrt{(1 \times 10^6/100)} = 100\ rad/s$

Frequency ratio:

$r = 628.32/100 = 6.2832$

Damping coefficient:

$c = 2\zeta\sqrt{(km)} = 2(0.3)\sqrt{(10^6 \times 100)} = 6000\ N{\cdot}s/m$

Denominator for amplitude:

$k - m\omega^2 = 10^6 - 100(628.32)^2 = -3.8478 \times 10^7$

$c\omega = 6000(628.32) = 3.7699 \times 10^6$

$D = \sqrt{((-3.8478 \times 10^7)^2 + (3.7699 \times 10^6)^2)}$

$D \approx 3.866 \times 10^7$ N/m

Amplitude:

$X = 525 / (3.866 \times 10^7) = 1.358 \times 10^{-5}$ m

$X = 0.01358$ mm

Transmissibility:

$(2\zeta r)^2 = (2 \times 0.3 \times 6.2832)^2 = 14.212$

$(1 - r^2)^2 = (1 - 39.478)^2 = 1480.59$

$T_r = \sqrt{(1 + 14.212)} / \sqrt{(1480.59 + 14.212)} = 3.900/38.662 = 0.1009$

Transmitted force:

$F_T = T_r F_0 = 0.1009(525) = 52.97$ N

Final Answer:

$X \approx 1.358 \times 10^{-5}$ m = 0.0136 mm

$T_r \approx 0.101,\ F_T \approx 53.0$ N

Diagram Description: A mass-spring-damper system excited by a rotating unbalance force F_o = 525 N at 6000 rpm. The transmissibility curve shows r = 6.28 > √2, so the system operates in the isolation region with T_r < 1.

- **Q.** Numerical type – Engine force analysis (inertia force, thrust, torque). Thrust on connecting rod, side thrust, tangential force, bearing load at a crank angle (W24, W23). Inertia torque at a crank angle (S25). Slider-crank 150/600 mm, 60°, 450 rpm: velocity and acceleration of piston, ω and α of rod (S23, S22 – same). **(Rank 8; Papers: 5 of 9)**

Answer:

Given:

- Crank radius, r = 150 mm = 0.15 m
- Connecting rod length, l = 600 mm = 0.60 m
- Crank angle, θ = 60°
- Speed, N = 450 rpm
- **To Find:** Velocity and acceleration of piston; angular velocity ω_c and angular acceleration α_c of connecting rod.

Formula:

$\omega = 2\pi N/60,\ \sin\varphi = r \sin\theta / l$

$v = r\omega [\sin\theta + r \sin\theta \cos\theta / (l \cos\varphi)]$

$a = r\omega^2 [\cos\theta + r \cos 2\theta / (l \cos\varphi)]$

$\omega_c = r\omega \cos\theta / (l \cos\varphi),\ \alpha_c = [-(r\omega^2 \sin\theta / l) + \sin\varphi\ \omega_c^2] / \cos\varphi$

Solution:

Angular speed of crank:

$\omega = 2\pi(450)/60 = 47.124$ rad/s

Obliquity:

$\sin\varphi = 0.15 \sin 60° / 0.60 = 0.2165$

$\varphi = 12.50°,\ \cos\varphi = 0.9763$

Piston velocity:

$v = 0.15(47.124)[0.8660 + 0.15(0.8660)(0.5) / (0.60(0.9763))]$

$v = 7.0686[0.8660 + 0.1109]$

$v = 6.91$ m/s

Piston acceleration:

$$a = 0.15(47.124)^2[0.5 + 0.25(-0.5)/0.9763]$$

$$a = 333.10[0.5 - 0.1280]$$

$$a = 123.9\ m/s^2$$

Angular velocity of connecting rod:

$$\omega_c = 0.15(47.124)(0.5) / (0.60(0.9763)) = 6.03\ rad/s$$

Angular acceleration of connecting rod:

$$\alpha_c = [-0.15(47.124)^2(0.8660)/0.60 + (0.2165)(6.03)^2] / 0.9763$$

$$\alpha_c = -484.4\ rad/s^2$$

Final Answer:

$$v_{piston} \approx 6.91\ m/s,\ a_{piston} \approx 123.9\ m/s^2$$

$$\omega_c \approx 6.03\ rad/s,\ \alpha_c \approx 484\ rad/s^2 \text{ opposite to } \omega_c$$

Diagram Description: A slider-crank mechanism with crank radius r = 150 mm, connecting rod l = 600 mm, crank angle 60°, and piston reciprocating along the line of stroke.

Engine Force Analysis — Formula Bank for Numerical

- **Gas force:** $F_g = pA$
- **Inertia force:** $F_I = m r \omega^2 (\cos\theta + (r/l) \cos 2\theta)$
- **Piston effort:** $F_p = F_g \pm F_I$
- **Force along connecting rod:** $F_c = F_p / \cos\phi$
- **Side thrust on cylinder wall:** $F_N = F_p \tan\phi$
- **Crank effort / tangential force:** $F_t = F_p \sin(\theta + \phi) / \cos\phi$
- **Turning moment:** $T = F_t r$

- **Q.** Numerical type – Torsional vibration (two / three rotor, equivalent shaft, node). Two rotors on stepped shaft, node and frequency (S22, W25). Motor-pump with gearing (S26). Equivalent length of shaft (W23). **(Rank 9; Papers: 4 of 9)**

Answer:

Given:

- Shaft diameter, d = 50 mm = 0.05 m
- Length, L = 1.5 m
- Shear modulus, G = 80 GPa = 80×10^9 N/m²
- Rotor inertias: $I_1 = 10$ kg·m², $I_2 = 15$ kg·m²
- **To Find:** Natural frequency of torsional vibration and node position.

Formula:

$$J = \pi d^4/32,\ k_t = GJ/L$$

$$\omega_n = \sqrt{[k_t(I_1 + I_2)/(I_1 I_2)]}$$

$$l_1 = [I_2/(I_1 + I_2)]\ L$$

Solution:

Polar moment of inertia:

$$J = \pi(0.05)^4/32 = \pi(6.25 \times 10^{-6})/32 = 6.136 \times 10^{-7}\ m^4$$

Torsional stiffness:

$$k_t = 80 \times 10^9 \times 6.136 \times 10^{-7} / 1.5$$

$$k_t = 49{,}088/1.5 = 32{,}725\ N\cdot m/rad$$

Natural frequency:

$$\omega_n = \sqrt{[32725(10 + 15)/(10 \times 15)]}$$

$\omega_n = \sqrt{[32725(25)/150]} = \sqrt{5454.17} = 73.85$ rad/s

$f_n = 73.85/2\pi = 11.75$ Hz

Node position from rotor 1:

$l_1 = [15/(10 + 15)](1.5) = (15/25)(1.5) = 0.9$ m

Node is 0.9 m from rotor 1, 0.6 m from rotor 2.

Final Answer:

$\omega_n = 73.85$ rad/s, $f_n = 11.75$ Hz

Node at 0.9 m from rotor 1 (or 0.6 m from rotor 2)

- **Q.** Numerical type – Locomotive and multi-cylinder engine balancing. Locomotive: balance masses and hammer blow / wheel lift-off speed (S26, S25, S23). Four-crank engine primary balance + secondary force (W24, S24); six-cylinder in-line (W23); V-engine secondary force (S26 OR). **(Rank 10; Papers: 4–5 of 9)**

Answer:

Given:

- Reciprocating mass per cylinder, m = 300 kg
- Crank radius, r = 0.3 m
- Balancing mass per cylinder, m_b = 150 kg at crank radius
- Speed, N = 300 rpm
- Distance between cylinder lines, a = 1.5 m
- **To Find:** Hammer blow, maximum tractive force, maximum swaying couple.

Formula:

$c = m_b/m$, $\omega = 2\pi N/60$

Hammer blow = $cmr\omega^2$

$F_{t,max} = (1 - c)\, mr\omega^2$

$C_{s,max} = (1 - c)\, mr\omega^2\, a$

Solution:

Balance factor:

$c = 150/300 = 0.5$

Angular speed:

$\omega = 2\pi(300)/60 = 31.416$ rad/s

Hammer blow:

$F_h = 0.5(300)(0.3)(31.416)^2$

$F_h = 0.5(300)(0.3)(987.0) = 44{,}415$ N

Maximum tractive force:

$F_{t,max} = (1 - 0.5)(300)(0.3)(987.0) = 44{,}415$ N

Maximum swaying couple:

$C_{s,max} = 44{,}415(1.5) = 66{,}622.5$ N·m

Final Answer:

Hammer blow ≈ 44.4 kN

$F_{t,max} \approx 44.4$ kN, $C_{s,max} \approx 66.6$ kN·m

Diagram Description: A locomotive with two cylinders and coupled wheels. The balancing mass on each crank produces a vertical hammer blow and horizontal tractive force; the offset a between cylinder lines produces the swaying couple.

Chapter 1 – Dynamic Force Analysis of Mechanisms

1. ★★★ 5+ papers – prepare FIRST:

- **Q.** State D'Alembert's principle and explain its use / application. **(Priority ★★★; Asked in: W23, W22, S26, S25, S23, S22; 3 / 4 Marks)**

Answer:

- **D'Alembert's principle:** A moving body can be brought into static equilibrium by adding an inertia force equal to the product of mass and acceleration, acting opposite to acceleration, and an inertia couple opposite to angular acceleration.
- **Force condition:** $\Sigma F + F_i = 0$, where $F_i = -ma$
- **Couple condition:** $\Sigma T + T_i = 0$, where $T_i = -I\alpha$
- **Use:** Converts a dynamic problem into a static equilibrium problem.
- **Application:** Force analysis of slider-crank mechanism, reciprocating engine parts, bearing reactions, balancing, and frame forces.

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2. ★★ Asked in 3–4 papers:

- **Q.** Difference between piston effort, crank effort and crank-pin effort; derive turning moment in terms of piston effort. **(Priority ★★; Asked in: W22, S23, S24; 4 / 7 Marks)**

Answer:

Term	Meaning	Formula / Direction
Piston effort F_p	Net force on piston along line of stroke	F_p = F_g ± F_I; along cylinder axis
Crank-pin effort F_c	Force transmitted through connecting rod to crankpin	F_c = F_p / cosϕ; along connecting rod
Crank effort F_t	Tangential component of crank-pin effort at crank radius	F_t = F_c sin(θ + ϕ) = F_p sin(θ + ϕ) / cosϕ

Derivation of turning moment:

Let F_p = piston effort, r = crank radius, l = connecting rod length, θ = crank angle from IDC, ϕ = obliquity of connecting rod.

Force along connecting rod:

$$F_c = F_p / \cos\varphi$$

Tangential force at crankpin:

$$F_t = F_c \sin(\theta + \varphi) = F_p \sin(\theta + \varphi) / \cos\varphi$$

Turning moment on crankshaft:

$$T = F_t r = F_p r \sin(\theta + \varphi) / \cos\varphi$$

Using $\sin(\theta + \phi)/\cos\phi = \sin\theta + \cos\theta \tan\phi$ and $\tan\phi \approx (r/l)\sin\theta$:

$$T \approx F_p r [\sin\theta + (r/2l) \sin 2\theta]$$

Diagram Description: A slider-crank mechanism with crank radius r, connecting rod length l, crank angle ϑ, and connecting-rod obliquity φ. Piston effort acts along the line of stroke; crank effort acts tangentially at the crankpin.

- **Q.** Dynamically equivalent system of two masses (requirements / conditions). **(Priority ★★; Asked in: W22, S24; 3 / 4 Marks)**

Answer:

- **Dynamically equivalent system:** A system of two point masses which has the same dynamic effect as the original rigid body.

For a rigid body of mass m, centre of gravity G, and mass moment of inertia I_G, replaced by masses m_1 and m_2 at distances l_1 and l_2 from G:

- **Total mass equal:**

  $m_1 + m_2 = m$
- **Centre of gravity same:**

  $m_1 l_1 = m_2 l_2$
- **Mass moment of inertia same:**

  $m_1 l_1^2 + m_2 l_2^2 = I_G$

For a connecting rod replaced by masses at piston pin and crankpin, all three conditions generally cannot be satisfied exactly; hence a correction couple is applied.

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3. ★ Asked 1–2 times – only if time is left:

- **Q.** Define inertia force and inertia couple. **(Priority ★; Asked in: W22, S22; 3 Marks)**

Answer:

- **Inertia force:** A fictitious force equal and opposite to the accelerating force on a body.

  $F_i = -ma$
- **Inertia couple:** A fictitious couple equal and opposite to the applied torque causing angular acceleration.

  $T_i = -I\alpha$

Used with D'Alembert's principle to convert dynamic equilibrium into static equilibrium.

- **Q.** Significance of inertia forces in reciprocating engines; why correction couple is applied. **(Priority ★; Asked in: W24; 3 / 4 Marks)**

Answer:

- Reciprocating parts have variable acceleration; inertia forces vary with crank angle.
- At high speed, inertia forces become larger than gas forces and cause bearing loads, stresses, vibration, and noise.
- Inertia forces are essential in balancing, crankshaft design, and bearing reaction calculations.
- **Correction couple:** When a connecting rod is replaced by two point masses at piston pin and crankpin, the mass moment of inertia condition is not exactly satisfied; a correction couple is added:

  $C_corr = (I_actual - I_equiv)\ \alpha$

- **Q.** Lami's theorem with example (W25); why crank is shaped like a bulb (W23). **(Priority ★; Asked in: W25, W23; 3 Marks)**

Answer:

- **Lami's theorem:** If three coplanar concurrent forces are in equilibrium, each force is proportional to the sine of the angle between the other two forces:

  $P/\sin\alpha = Q/\sin\beta = R/\sin\gamma$

Diagram Description: A particle O in equilibrium under three concurrent forces P, Q, R. Angle α is between Q and R, β between R and P, and γ between P and Q.

- **Example:** A weight W = 100 N is suspended by two strings making 30° and 60° with the horizontal. By Lami's theorem, tensions are T_1 = 50 N and T_2 = 86.6 N.
- **Why crank is shaped like a bulb:** The bulb-shaped crank web provides counterweight mass at a larger radius to balance rotating/reciprocating masses, and the smooth enlarged boss reduces stress concentration and gives strength/stiffness near the crankpin.

Chapter 2 – Turning Moment Diagram and Flywheel

1. ★★★ 5+ papers – prepare FIRST:

- **Q.** Turning moment diagram of four-stroke IC engine and of multi-cylinder engine. **(Priority ★★★; Asked in: W23, S22, S23, S26; 3 / 4 Marks)**

Answer:

- **Turning moment diagram (TMD):** Graph of turning moment / crank effort torque vs crank angle for one complete cycle.

Four-stroke IC engine:

- One cycle = 2 revolutions of crank = 720°.
- Suction stroke: small negative turning moment.
- Compression stroke: negative turning moment, increasing in magnitude.
- Expansion / power stroke: large positive turning moment.
- Exhaust stroke: small negative turning moment.
- Net work done per cycle = area under TMD.

Multi-cylinder engine:

- TMDs of individual cylinders are superimposed with proper firing order and phase difference.
- More cylinders give more uniform turning moment and smaller flywheel.
- For a 4-cylinder 4-stroke engine, power strokes occur at intervals of 180°.
- Fluctuation of energy is reduced; engine runs smoother.

Diagram Description: A four-stroke engine TMD with negative loops for suction, compression, and exhaust, and a large positive loop for expansion over 720°. For a multi-cylinder engine, several positive loops overlap, giving a flatter mean torque line.

- **Q.** Define fluctuation of energy, fluctuation of speed, coefficient of fluctuation of speed and of energy. **(Priority ★★★; Asked in: W25, W22, W23, S23; 3 / 4 Marks)**

Answer:

- **Fluctuation of energy (ΔE):** Difference between maximum and minimum kinetic energy of the flywheel during a cycle.

  $\Delta E = E_{max} - E_{min}$
- **Fluctuation of speed:** Difference between maximum and minimum speeds during a cycle.

  $N_1 - N_2$ or $\omega_1 - \omega_2$
- **Coefficient of fluctuation of speed (C_s):**

  $C_s = (N_1 - N_2)/N = (\omega_1 - \omega_2)/\omega$

where N or ω is mean speed.

- **Coefficient of fluctuation of energy (C_e):** Ratio of maximum fluctuation of energy to work done per cycle.

  $C_e = \Delta E / W_{cycle}$
- **Flywheel energy relation:**

  $\Delta E = I\omega^2 C_s = mk^2\omega^2 C_s$

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2. ★★ Asked in 3–4 papers:

- **Q.** Role of flywheel in punching press. **(Priority ★★; Asked in: S26; 4 Marks)**

Answer:

- A punching press requires a large amount of energy in a very short time during the actual punching stroke.

- The motor is small and supplies energy uniformly over the whole cycle.
- The flywheel stores energy during the idle part of the cycle and releases it during punching.
- This reduces the required motor power and keeps the speed fluctuation within limits.

Energy supplied by flywheel:

$$\Delta E = E_punch - E_motor\ during\ punching$$

Flywheel mass is found from:

$$\Delta E = mk^2\omega^2 C_s$$

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3. ★ Asked 1–2 times – only if time is left:

- **Q.** Flywheel in two-wheeler and how it differs from governor. **(Priority ★; Asked in: S25; 4 Marks)**

Answer:

Flywheel in two-wheeler:

- Smooths out power pulses of the single/twin-cylinder engine.
- Stores energy during power stroke and releases it during other strokes.
- Reduces vibration and speed fluctuation within each cycle.
- Helps steady idle and prevents stalling.

Difference between flywheel and governor:

Basis	Flywheel	Governor
Function	Controls speed fluctuation within a cycle	Controls mean speed under load changes
Action	Stores and releases energy	Regulates fuel/steam supply
Type	Dynamic energy reservoir	Speed-sensitive control device
Speed variation	Reduces cyclic variation	Maintains nearly constant mean speed

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Chapter 3 – Balancing

1. ★★★ 5+ papers – prepare FIRST:

- **Q.** Static balancing vs dynamic balancing, with examples and conditions. **(Priority ★★★; Asked in: W25, S26, S25, S24, S23, S22; 3 / 4 Marks)**

Answer:

- **Static balancing:** The centre of gravity of all rotating masses lies on the axis of rotation; the vector sum of centrifugal forces is zero.

$$\Sigma mr = 0$$

Example: single disc, grinding wheel, ceiling fan blade.

- **Dynamic balancing:** Both the resultant centrifugal force and the resultant centrifugal couple are zero.

$$\Sigma mr = 0 \text{ and } \Sigma mrl = 0$$

Example: crankshaft, turbine rotor, multi-cylinder engine crankshaft.

A dynamically balanced system is always statically balanced, but a statically balanced system may not be dynamically balanced.

Basis	Static balancing	Dynamic balancing
Condition	Σmr = 0	Σmr = 0 and Σmrl = 0
Couple	Not considered	Couple balanced
Example	Grinding wheel	Crankshaft
Used for	Low-speed rotors	High-speed rotors

- **Q.** Why balancing of rotating parts is necessary, especially for high-speed machines; effects of unbalance. **(Priority ★★★; Asked in: W25, W23, W22, S22; 3 / 4 Marks)**

Answer:

- Unbalance produces centrifugal force $F = mr\omega^2$, which increases with the square of speed; at high speed it becomes very large.
- **Effects of unbalance:** vibration, noise, fatigue failure, bearing wear, shaft deflection, foundation damage, reduced machine life, and unsafe operation.
- Balancing reduces alternating forces and couples transmitted to bearings and frame.
- It is essential for smooth operation, accuracy, comfort, and safety in high-speed machines such as turbines, engines, compressors, and machine tools.

- **Q.** Balancing of V-engines; derive secondary unbalanced force; primary force magnitude and direction. **(Priority ★★★; Asked in: W23, W22, S24, S26; 4 / 7 Marks)**

Answer:

- **V-engine:** Two cylinders inclined at angle 2α on a common crank. Let m = reciprocating mass per cylinder, r = crank radius, l = connecting rod length, ω = angular speed, θ = crank angle from the bisector.

Primary force:

Primary force in each cylinder:

$$F_{p1} = mr\omega^2 \cos(\theta - \alpha),\ F_{p2} = mr\omega^2 \cos(\theta + \alpha)$$

Resolving along the bisector and perpendicular to it, the resultant primary force is

$$F_p = 2mr\omega^2 \cos\alpha$$

acting along the crank.

Secondary unbalanced force:

Secondary force in each cylinder:

$$F_{s1} = (mr^2\omega^2/l)\cos2(\theta - \alpha),\ F_{s2} = (mr^2\omega^2/l)\cos2(\theta + \alpha)$$

Resolving and summing components:

$$F_s = (2mr^2\omega^2/l)\cos\alpha\cos2\theta$$

- **Direction:** along the bisector of the two cylinder axes.

Diagram Description: A V-engine with two cylinders inclined symmetrically at angle 2α about the vertical bisector. The crank rotates at angle θ from the bisector; primary force acts along the crank and secondary force acts along the bisector.

- **Q.** Primary and secondary unbalanced forces – compare; imaginary crank; partial balancing of primary force; why only part of reciprocating force is balanced. **(Priority ★★★; Asked in: W24, W22, S26, S24, S22; 3 / 4 / 7 Marks)**

Answer:

- **Primary unbalanced force:**

$$F_p = mr\omega^2\cos\theta$$

Frequency = once per revolution.

- **Secondary unbalanced force:**

$$F_s = (mr^2\omega^2/l)\cos2\theta$$

Frequency = twice per revolution; magnitude smaller by factor r/l.

- **Imaginary crank:** The secondary force is represented by a fictitious crank of length $r^2/(4l)$ rotating at 2ω; used to draw the secondary force polygon.
- **Partial balancing:** Only a fraction c of the reciprocating mass is balanced by a counterweight.

$$c = \text{balancing mass} / \text{reciprocating mass}$$

Typically c = 0.5 to 0.67.

- **Why only part is balanced:** Complete balancing of primary force needs a large counterweight, which creates a large unbalanced force perpendicular to the line of stroke. Partial balancing reduces the primary force along the line of stroke while keeping the perpendicular unbalanced force within limits.

Basis	Primary force	Secondary force
Expression	$mr\omega^2\cos\theta$	$(mr^2\omega^2/l)\cos2\theta$
Frequency	Once per revolution	Twice per revolution
Magnitude	Larger	Smaller by r/l
Balancing	By counterweight	By imaginary crank / special arrangement

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2. ★★ Asked in 3–4 papers:

- **Q.** Locomotive terms: tractive force variation, swaying couple, hammer blow (derive). **(Priority ★★; Asked in: W25, W24, W23, S23; 3 / 4 / 7 Marks)**

Answer:

- **Tractive force:** Net horizontal force along the direction of motion.

$$F_t = (1 - c)\,mr\omega^2\cos\theta$$

It varies harmonically and causes fore-and-aft oscillation.

- **Swaying couple:** Unbalanced primary forces in the two cylinders produce a couple about the vertical axis.

$$C_s = (1 - c)\, mr\omega^2\, a \sin\theta$$

where a = distance between the lines of stroke of the two cylinders. It causes nosing of the locomotive.

- **Hammer blow:** The balancing mass cm at crank radius r produces a vertical force

$$F_v = cmr\omega^2 \sin\theta$$

Maximum vertical force:

$$\text{Hammer blow} = cmr\omega^2$$

It causes alternating load on the rails and track damage.

Diagram Description: A locomotive with two cylinders and coupled wheels. The balancing mass on the crank produces a vertical component (hammer blow) and horizontal component (tractive force); the offset between cylinder lines produces the swaying couple.

- **Q.** Balancing of inline engines – procedure. **(Priority ★★; Asked in: S25, S23; 4 / 7 Marks)**

Answer:

1. Draw the crank arrangement and firing order of the multi-cylinder engine.
2. Calculate primary and secondary forces and couples for each cylinder using $mr\omega^2$ and $mr^2\omega^2/l$.
3. Resolve all forces and couples along and perpendicular to the crank.
4. Sum the components; if the resultant force and couple are zero, the engine is balanced.
5. For unbalanced primary/secondary effects, add counterweights or use a balance shaft.
6. Verify using force polygon and couple polygon.

Typical results:

- **Four-cylinder inline:** primary forces and couples balance; secondary forces do not balance; secondary couple balances.
- **Six-cylinder inline:** primary and secondary forces and couples all balance.

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3. ★ Asked 1–2 times – only if time is left:

- **Q.** Why leading and trailing wheels of locomotive are coupled (S25); criteria for reference plane (S24); several masses in same plane (S22); three masses in different planes (S25). **(Priority ★; Asked in: S25, S24, S22; 3 / 4 Marks)**

Answer:

- **Why leading and trailing wheels of locomotive are coupled:** Coupling rods distribute the driving torque, increase adhesion weight, improve tractive effort, and reduce wheel slip.
- **Criteria for reference plane:** Any convenient plane may be chosen; distances are measured from it. The force polygon is independent of the reference plane, while the couple polygon depends on it.
- **Several masses in the same plane:** Only static balancing is required; $\Sigma mr = 0$; the force polygon closes.
- **Three masses in different planes:** Dynamic balancing is required; both $\Sigma mr = 0$ and $\Sigma mrl = 0$; force and couple polygons must close.

Chapter 4 – Gyroscope

1. ★★★ 5+ papers – prepare FIRST:

- **Q.** Derive gyroscopic couple formula C = Iωωp. **(Priority ★★★; Asked in: S23, S22; 3 / 4 Marks)**

Answer:

- **Gyroscopic couple:** The couple produced on a rotating body when its axis of spin is forced to precess.

Let a rotor have mass m, radius of gyration k, and spin angular velocity ω.

$$I = mk^2, \; L = I\omega$$

Let the axis of spin precess with angular velocity ω_p. In time dt, the axis turns through angle

$$d\theta = \omega_p \, dt$$

The angular momentum vector L changes direction; its magnitude remains constant. The change in angular momentum is

$$dL = L \, d\theta = I\omega \, \omega_p \, dt$$

Gyroscopic couple is the rate of change of angular momentum:

$$C = dL/dt = I\omega \, \omega_p$$

$$C = I\omega\omega_p$$

- **Direction:** By right-hand rule, the gyroscopic couple acts in a plane perpendicular to both the axis of spin and the axis of precession.

Diagram Description: A rotor spinning about axis OX with angular velocity ω, while OX itself rotates about axis OY with precession ω_p. The angular momentum vector L sweeps a cone; the change dL gives the gyroscopic couple C.

- **Q.** Effect of gyroscopic couple on ship during steering, pitching and rolling; define steering, pitching, rolling. **(Priority ★★★; Asked in: W25, W22, S23; 3 / 4 Marks)**

Answer:

Definitions:

- **Steering:** Turning of the ship about a vertical axis (yaw) to change course.
- **Pitching:** Oscillation of the ship about a transverse axis; bow rises and stern sinks alternately.
- **Rolling:** Oscillation of the ship about a longitudinal axis; side-to-side tilt.

Gyroscopic couple:

$$C = I\omega\omega_p$$

1. Effect during steering:

Precession angular velocity:

$$\omega_p = v/R$$

where v = ship speed, R = turning radius.

- The gyroscopic couple acts in a vertical plane.
- It tends to raise the bow and lower the stern, or vice versa, depending on the direction of rotor rotation and turn.
- This causes a dipping/raising effect on the ship.

2. Effect during pitching:

- The axis of spin precesses about a transverse axis.
- The gyroscopic couple acts about a vertical axis.
- It tends to turn the ship in yaw — the bow swings toward port or starboard.
- This is undesirable for steering stability.

3. Effect during rolling:

- The axis of spin is longitudinal, the same as the axis of rolling, so the spin axis does not precess.
- Since the axis of precession is parallel to the axis of spin, ω_p = 0 and no gyroscopic couple acts.
- Hence rolling has no gyroscopic effect on the ship.

Motion	Axis of precession	Plane of gyroscopic couple	Effect
Steering	Vertical	Vertical plane	Bow rises / dips
Pitching	Transverse	Horizontal plane	Yaw / turning of bow
Rolling	Longitudinal (parallel to spin axis)	None	No gyroscopic effect

Diagram Description: A ship with a turbine rotor spinning about a longitudinal axis. Arrows show precession about vertical, transverse, and longitudinal axes for steering, pitching, and rolling respectively; the resulting gyroscopic couple acts perpendicular to both spin and precession axes.

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2. ★★ Asked in 3–4 papers:

- **Q.** Effect on aeroplane while taking a turn (nose up / down). **(Priority ★★; Asked in: W23, W22, S25; 4 Marks)**

Answer:

- The propeller rotates about the longitudinal axis of the aeroplane.
- When the aeroplane takes a turn, the axis of spin precesses about a vertical axis.
- The gyroscopic couple acts about a transverse axis, causing the nose to rise or dip.
- Direction depends on the direction of propeller rotation and the direction of turn.
- For a clockwise propeller viewed from the pilot and a left turn, the nose tends to rise; for a right turn, the nose tends to dip (or vice versa depending on convention).
- The pilot must correct this tendency using elevators.

Diagram Description: An aeroplane in a horizontal turn with propeller spinning about the longitudinal axis. The precession axis is vertical; the gyroscopic couple acts about the transverse axis, producing a nose-up or nose-down tendency.

- **Q.** Active and reactive gyroscopic couple; define axis of spin, plane of precession, axis of gyroscopic reaction. **(Priority ★★; Asked in: W25, W23; 3 / 4 Marks)**

Answer:

- **Active gyroscopic couple:** The external couple applied to the axis of spin to cause precession.
- **Reactive gyroscopic couple:** The equal and opposite couple exerted by the rotating body on its supports; it is the reaction to the active couple.
- **Axis of spin:** The axis about which the rotor rotates.
- **Plane of precession:** The plane in which the axis of spin rotates.
- **Axis of gyroscopic reaction:** The axis about which the reactive gyroscopic couple acts; perpendicular to both spin axis and precession axis.
- **Relation:**

  $C_{active} = C_{reactive} = I\omega\omega_p$

- **Q.** Effect of gyroscopic couple on stability of two-wheeler and on four wheels of vehicle on a curve. **(Priority ★★; Asked in: S26, S22, S23; 3 / 7 Marks)**

Answer:

Two-wheeler:

- The wheels rotate about a transverse axis.
- When the vehicle takes a turn, the wheels precess about a vertical axis.
- The gyroscopic couple acts about the longitudinal axis and helps the rider to lean and balance.
- At higher speeds, the gyroscopic effect increases ($C \propto \omega\omega_p$) and improves stability.
- The rider leans inward to balance the centrifugal moment and the gyroscopic couple.

Four-wheeler on a curve:

- The rotating wheels and engine parts produce a gyroscopic couple when the vehicle turns.
- The couple tends to transfer load from inner wheels to outer wheels (or vice versa).
- It may cause wheel lift-off at high speed or sharp turns.

Magnitude:

$$C = I\omega\omega_p, \ \omega_p = v/R$$

The load transfer due to gyroscopic couple is

$$\Delta W = C/t$$

where t = track width.

Diagram Description: A two-wheeler leaning into a turn, with wheel spin axis transverse and precession axis vertical; the gyroscopic couple acts along the longitudinal axis. For a four-wheeler, the gyroscopic couple acts about the longitudinal axis and changes normal reactions on inner and outer wheels.

Chapter 5 – Free and Damped Free Vibrations

1. ★★★ 5+ papers – prepare FIRST:

- **Q.** Define natural frequency, damping, free / forced vibration, resonance, period, cycle, frequency. **(Priority ★★★; Asked in: W25, W24, W23, W22, S25, S23, S22; 3 / 4 Marks)**

Answer:

- **Natural frequency (f_n or ω_n):** The frequency at which a system oscillates freely after being disturbed from equilibrium, without external forcing.
  $\omega_n = \sqrt{(k/m)}$ or $\sqrt{(g/\delta)}$
- **Damping:** The resistance to motion that dissipates vibration energy, reducing amplitude over time.
- **Free vibration:** Vibration of a system under its own internal restoring force after an initial disturbance, with no external excitation.
- **Forced vibration:** Vibration produced by a continuous external periodic force.
- **Resonance:** Condition when excitation frequency equals natural frequency (ω = ω_n); amplitude becomes very large (limited only by damping).
- **Period (T):** Time for one complete cycle.
  $T = 2\pi/\omega_n = 1/f_n$
- **Cycle:** One complete sequence of motion from a given state back to the same state.
- **Frequency (f_n):** Number of cycles per second (Hz).
  $f_n = 1/T = \omega_n/2\pi$

- **Q.** Logarithmic decrement – define, derive expression, use. **(Priority ★★★; Asked in: W25, W24, W22, S26, S22, S24; 3 / 4 Marks)**

Answer:

- **Logarithmic decrement (δ):** Natural logarithm of the ratio of two successive amplitudes of a damped free vibration.
  $\delta = \ln(x_1/x_2)$

Derivation:

For under-damped free vibration:

$x = X\, e^{(-\zeta\omega_n t)} \sin(\omega_d t + \varphi)$

At times t_1 and $t_2 = t_1 + T_d$, where $T_d = 2\pi/\omega_d$:

$x_1 = X\, e^{(-\zeta\omega_n t_1)} \sin(\omega_d t_1 + \varphi)$

$x_2 = X\, e^{(-\zeta\omega_n (t_1 + T_d))} \sin(\omega_d t_1 + \varphi)$

Taking ratio:

$x_1/x_2 = e^{(\zeta\omega_n T_d)}$

Therefore:

$\delta = \ln(x_1/x_2) = \zeta\omega_n T_d$

Substituting $T_d = 2\pi / (\omega_n \sqrt{(1 - \zeta^2)})$:

$\delta = 2\pi\zeta / \sqrt{(1 - \zeta^2)}$

- **Use:** Determines damping factor ζ from measured successive amplitudes:
  $\zeta = \delta / \sqrt{((2\pi)^2 + \delta^2)}$

- **Q.** Under-damped, critically damped, over-damped systems – compare curves; derive equation for critically damped system. **(Priority ★★★; Asked in: W25, W24, W22, S25, S24, S23; 3 / 4 Marks)**

Answer:

Derivation of critical damping condition:

Equation of free damped vibration:

$$m\ddot{x} + c\dot{x} + kx = 0$$

Characteristic equation:

$$ms^2 + cs + k = 0$$

Roots:

$$s_{1,2} = [-c \pm \sqrt{(c^2 - 4mk)}] / 2m$$

Critical damping occurs when discriminant is zero:

$$c^2 = 4mk$$

$$c_c = 2\sqrt{(km)} = 2m\omega_n$$

Critical damping factor:

$$\zeta = c/c_c = 1$$

For critically damped system, repeated root $s = -\omega_n$, solution:

$$x = (A + Bt)\, e^{\wedge}(-\omega_n\, t)$$

Comparison:

System	Damping condition	Motion character	Amplitude decay
Under-damped	$c < c_c$ ($\zeta < 1$)	Oscillatory, decaying	Oscillations with decreasing amplitude
Critically damped	$c = c_c$ ($\zeta = 1$)	Non-oscillatory, fastest return to equilibrium	No oscillation
Over-damped	$c > c_c$ ($\zeta > 1$)	Non-oscillatory, slow return	No oscillation, sluggish

Diagram Description: Three curves starting from initial displacement: under-damped shows decaying oscillations crossing the equilibrium line; critically damped returns to zero without crossing; over-damped returns to zero more slowly without crossing.

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2. ★★ Asked in 3–4 papers:

- **Q.** Types of vibration – longitudinal, transverse, torsional (with sketches). **(Priority ★★; Asked in: W25, W24, W22, S24; 3 / 4 Marks)**

Answer:

- **Longitudinal vibration:** Particles oscillate along the length of the member; alternating push-pull. Example: spring-mass system, bar under axial load.
- **Transverse vibration:** Particles oscillate perpendicular to the length of the member. Example: cantilever beam, shaft with disc, string.
- **Torsional vibration:** Particles oscillate about the longitudinal axis; alternating twist. Example: shaft with rotors, crankshaft.

Diagram Description:

- **Longitudinal:** A bar/spring with arrows showing extension and compression along its axis.
- **Transverse:** A beam fixed at one end with the free end moving up and down.
- **Torsional:** A shaft with discs at ends, arrows showing twist about the axis.

- **Q.** Methods to find natural frequency – energy method, Rayleigh's method. **(Priority ★★; Asked in: W25, W24, S24; 3 / 4 Marks)**

Answer:

- **Energy method:** For conservative system, total energy (kinetic + potential) remains constant.

$$T_{max} = V_{max}$$

At equilibrium position, potential energy is zero and kinetic energy is maximum; at extreme position, kinetic energy is zero and potential energy is maximum. Solve for ω_n.

- **Rayleigh's method:** A special case of energy method for systems with approximate mode shape.

$$\omega_n = \sqrt{(g \Sigma m_i y_i / \Sigma m_i y_i^2)}$$

or for continuous systems, equate maximum kinetic energy to maximum potential energy using an assumed deflection curve.

Both methods give natural frequency without solving differential equation; Rayleigh's method requires an assumed deflection shape.

- **Q.** Types of damping / ways of providing damping; dry friction damping. **(Priority ★★; Asked in: W23, W22, S25, S23; 3 / 4 Marks)**

Answer:

- **Viscous damping:** Resistance proportional to velocity; provided by dashpot, fluid film.
- **Dry friction (Coulomb) damping:** Constant friction force opposing motion; provided by dry surfaces.
- **Structural / hysteretic damping:** Energy loss due to internal friction in material.
- **Air damping:** Resistance due to air/viscous fluid.

Dry friction damping:

$$F_d = \mu N$$

Amplitude decreases linearly with time; motion stops after finite time. Used in vehicle suspensions, brake linings.

- **Q.** Torsionally equivalent shaft; torsional vibration of one, two and three rotor systems. **(Priority ★★; Asked in: W22, W23, S23, S22; 4 / 7 Marks)**

Answer:

- **Torsionally equivalent shaft:** A single shaft of uniform diameter and equivalent length L_e that has the same torsional stiffness as a stepped shaft.

$$L_e = L_1(d/d_1)^4 + L_2(d/d_2)^4 + \cdots$$

- **One-rotor system:** Shaft fixed at one end, rotor at other.

$$\omega_n = \sqrt{(GJ/(IL))}$$

where G = shear modulus, J = polar moment of inertia, I = mass moment of inertia of rotor.

- **Two-rotor system:** Two rotors on a shaft, free-free system. Node lies between them.

$$\omega_n = \sqrt{[(GJ/L)((I_1 + I_2)/(I_1 I_2))]}$$

Distance of node from rotor 1:

$$l_1 = [I_2/(I_1 + I_2)]\, L$$

- **Three-rotor system:** For rotors I_1, I_2, I_3 with shaft segments L_{12}, L_{23}:
- Write torsional equations for each rotor; set up frequency equation and solve for ω_n.
- Two natural frequencies typically exist.
- Node locations determined from equilibrium of torsional moments.

Diagram Description: A shaft with rotors at intervals; node points indicated where amplitude is zero. For two rotors, the node divides the shaft in inverse ratio of inertias.

3. ★ Asked 1–2 times – only if time is left:

- **Q.** Effect of inertia of shaft (W24); causes and effects of vibration (W25). **(Priority ★; Asked in: W24, W25; 3 Marks)**

Answer:

- **Inertia of shaft:** In torsional vibration, shaft mass moment of inertia adds to rotor inertia, reducing natural frequency:

  $\omega_n = \sqrt{[k_t / (I_rotor + I_shaft/3)]}$

(For uniform shaft, effective shaft inertia is one-third about fixed end.)

- **Causes of vibration:** Unbalance, misalignment, looseness, worn bearings, aerodynamic/fluid forces, reciprocating forces, earthquakes, road roughness.
- **Effects of vibration:** Noise, fatigue failure, bearing wear, loosening of fasteners, poor surface finish, discomfort, structural damage.

Chapter 6 – Forced Damped Vibrations

1. ★★★ 5+ papers – prepare FIRST:

- **Q.** Vibration isolation and force transmissibility; transmissibility curve with regions of control. **(Priority ★★★; Asked in: W23, W22, S26, S25, S23, S22; 3 / 4 Marks)**

Answer:

- **Vibration isolation:** Reducing the force transmitted from a machine to its foundation (force isolation) or reducing motion transmitted to sensitive equipment (motion isolation).
- **Force transmissibility (T_r):** Ratio of transmitted force amplitude to exciting force amplitude.

$$T_r = F_T/F_0 = \sqrt{[(1 + (2\zeta r)^2) / ((1 - r^2)^2 + (2\zeta r)^2)]}$$

where r = ω/ω_n.

- **Isolation condition:** T_r < 1 when r > √2; practical design uses r = 2.5 to 5.

Regions of control:

Region	Condition	Controlled by
Amplification	r < √2	Stiffness
Resonance	r ≈ 1	Damping
Isolation	r > √2	Mass

Diagram Description: A transmissibility curve plotted against frequency ratio r. The curve starts at T_r = 1, peaks near r = 1, crosses T_r = 1 at r = √2, and then decreases for r > √2, entering the isolation region.

- **Q.** Magnification factor (dynamic magnifier) – define, expression, role; frequency ratio. **(Priority ★★★; Asked in: W25, W24, W22, S25, S24, S23, S22; 3 / 4 Marks)**

Answer:

- **Magnification factor (M):** Ratio of dynamic amplitude to static deflection under the same force.

$$M = X/(F_0/k) = 1 / \sqrt{[(1 - r^2)^2 + (2\zeta r)^2]}$$

- **Frequency ratio:**

$$r = \omega/\omega_n$$

- **Role:** Indicates amplification of vibration; at resonance r = 1,

$$M_max = 1/(2\zeta)$$

- **Effect:** Damping reduces M near resonance; for r ≫ 1, M → 0.

---

2. ★★ Asked in 3–4 papers:

- **Q.** Forced vibration due to rotating unbalance; vector / graphical representation of forced damped vibration; effect of damping on amplitude. **(Priority ★★; Asked in: W24, W22, S26, S24, S22; 3 / 4 / 7 Marks)**

Answer:

Let unbalance mass m_e rotate at radius e with angular speed ω.

Excitation force amplitude:

$$F_0 = m_e\ e\ \omega^2$$

Equation of motion:

$$m\ddot{x} + c\dot{x} + kx = m_e\ e\ \omega^2 \sin\omega t$$

Steady-state amplitude:

$$X = m_e\ e\ \omega^2 / \sqrt{[(k - m\omega^2)^2 + (c\omega)^2]}$$

Phase lag:

$\varphi = \tan^{-1}[c\omega / (k - m\omega^2)]$

- **Vector diagram:** The exciting force F_o, spring force kX, damping force cωX, and inertia force $m\omega^2X$ form a closed phasor polygon.
- kX and $m\omega^2X$ are opposite along displacement.
- cωX is perpendicular to displacement.
- F_o closes the polygon.

Effect of damping:

- Reduces amplitude near resonance.
- Limits maximum amplitude to $F_o/(2\zeta k)$.
- Shifts peak amplitude slightly below r = 1.
- At high r, damping increases transmissibility; hence low damping is preferred for isolation.

Diagram Description: A mass-spring-damper system with a rotating unbalance mass m_e at eccentricity e. The phasor diagram shows F_o, kX, cωX, and $m\omega^2X$ forming a closed vector polygon.

---

3. ★ Asked 1–2 times – only if time is left:

- **Q.** Materials used for isolation and their uses. **(Priority ★; Asked in: W23; 4 Marks)**

Answer:

Material	Property	Use
Rubber / Neoprene	High damping, good for high frequency	Machine mounts, instrument pads
Cork	Moderate damping, cheap	Foundation isolation, compressor pads
Felt	Low cost, moderate isolation	Light machinery, instrument bases
Steel springs	Low damping, large deflection	Heavy machines, vehicle suspension

- Elastomers are used where both isolation and damping are needed.
- Steel springs are used where large static deflection and low natural frequency are required.
- Cork/felt are used for small machines and impact isolation.

Chapter 7 – Critical Speeds of Shafts

1. ★★★ 5+ papers – prepare FIRST:

- **Q.** Derive critical (whirling) speed of shaft with single rotor, without damping. **(Priority ★★★; Asked in: W25, W23, W22, S25; 4 / 7 Marks)**

Answer:

- **Critical speed:** The speed at which the shaft begins to whirl violently due to resonance between the rotational speed and the natural frequency of transverse vibration.

Derivation (single rotor at mid-span, simply supported shaft):

Let a shaft of length L, Young's modulus E, and moment of inertia I carry a rotor of mass m at the centre. Let the static deflection at the centre be δ.

For a simply supported beam with central load W = mg, the central deflection is

$$\delta = WL^3/(48EI) = mgL^3/(48EI)$$

The equivalent spring stiffness at the rotor is

$$k = W/\delta = 48EI/L^3$$

The natural frequency of transverse vibration is

$$\omega_n = \sqrt{(k/m)} = \sqrt{(48EI/(mL^3))}$$

When the shaft rotates at angular speed ω, the centrifugal force due to the rotor's eccentricity causes whirling. Resonance occurs when

$$\omega = \omega_n$$

Hence the critical (whirling) speed is

$$\omega_c = \omega_n = \sqrt{(48EI/(mL^3))}$$

or in terms of static deflection:

$$\omega_c = \sqrt{(g/\delta)}$$

Diagram Description: A simply supported horizontal shaft of length L with a rotor of mass m at the centre. The shaft deflects by δ under the weight; the rotor whirls about the shaft axis with eccentricity e.

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2. ★★ Asked in 3–4 papers:

- **Q.** Explain whirling speed; prove whirling speed = natural frequency of transverse vibration. **(Priority ★★; Asked in: W25, W23, S24; 3 / 7 Marks)**

Answer:

- **Whirling speed:** The speed at which a rotating shaft tends to bow out and rotate about the bearing axis rather than its own axis, due to centrifugal force from unbalance. Also called critical speed.

Proof:

Consider a shaft with a rotor of mass m at mid-span, rotating at angular speed ω. Let the eccentricity of the centre of gravity from the geometric centre be e.

Let y be the additional deflection of the shaft due to centrifugal force. The total radial distance of the centre of gravity from the bearing axis is y + e.

Centrifugal force:

$$F_c = m\omega^2(y + e)$$

Restoring elastic force:

$$F_s = ky$$

For equilibrium:

$$m\omega^2(y + e) = ky$$

$$y = m\omega^2 e / (k - m\omega^2)$$

Deflection y becomes infinite when

$$k - m\omega^2 = 0$$

$$\omega = \sqrt{(k/m)} = \omega_n$$

Therefore, the whirling speed equals the natural frequency of transverse vibration:

$$\omega_c = \omega_n = \sqrt{(k/m)}$$

- **Q.** Dunkerley's method – derive / demonstrate for several loads. **(Priority ★★; Asked in: W25, S24; 4 / 7 Marks)**

Answer:

- **Dunkerley's method:** An approximate method to find the fundamental critical speed of a shaft carrying several loads.

Derivation:

Let a shaft carry loads W_1, W_2, …, W_n at different positions. Let δ_1, δ_2, …, δ_n be the static deflections at these loads due to each load acting alone.

The fundamental natural frequency is approximated by

$$1/\omega_c^2 \approx 1/\omega_1^2 + 1/\omega_2^2 + \cdots + 1/\omega_n^2$$

where ω_i is the critical speed due to load W_i acting alone.

Since ω_i = √(g/δ_i),

$$1/\omega_c^2 = \delta_1/g + \delta_2/g + \cdots + \delta_n/g$$

Hence

$$\omega_c = \sqrt{(g / (\delta_1 + \delta_2 + \cdots + \delta_n))}$$

Dunkerley's method always gives a lower bound to the true critical speed.

- **Demonstration for two loads:** For loads W_1 and W_2 at distances a and b from one end of a simply supported shaft of length L:

$$\delta_1 = W_1 a^2 (L - a)^2 / (3EIL), \quad \delta_2 = W_2 b^2 (L - b)^2 / (3EIL)$$

Then

$$\omega_c = \sqrt{(g / (\delta_1 + \delta_2))}$$

Diagram Description: A simply supported shaft carrying two discs at distances a and b from the left bearing. Deflection curves for each load acting alone are sketched; the sum of deflections gives the Dunkerley approximation.

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3. ★ Asked 1–2 times – only if time is left:

- **Q.** Effect of damping on critical speed; safety near critical speed; design changes to avoid resonance (S26); half-frequency whirl (W25); why soldiers break step on bridges (S25). **(Priority ★; Asked in: S26, W25, S25; 3 / 4 / 7 Marks)**

Answer:

- **Effect of damping:** Damping reduces the amplitude of whirl near critical speed but does not change the critical speed significantly. It increases the safe operating range near resonance.
- **Safety near critical speed:** Shafts should be operated either well below or well above critical speed; rapid passage through critical speed is recommended to limit amplitude buildup.
- **Design changes to avoid resonance:** Change shaft stiffness (diameter, length), change mass distribution, add intermediate bearings, use dampers, or alter operating speed.
- **Half-frequency whirl:** A self-excited instability in journal bearings where the shaft whirls at about half the rotational speed; caused by oil-film forces; can be destructive.
- **Why soldiers break step on bridges:** To avoid exciting the bridge's natural frequency by synchronized marching, which could cause resonant vibration and structural failure.

Chapter 8 – Vibration Monitoring and Analysis

1. ★★ Asked in 3–4 papers:

- **Q.** Vibrometer and accelerometer – working principle with sketch; classify vibration measuring instruments. **(Priority ★★; Asked in: S22, S25, W22, S23; 3 / 4 Marks)**

Answer:

Classify vibration measuring instruments:

By measured quantity:

- **Vibrometer (seismometer):** measures displacement amplitude of vibrating body.
- **Velometer:** measures velocity of vibration.
- **Accelerometer:** measures acceleration of vibration.

By principle:

- **Seismic instruments:** use a mass-spring-damper system with relative motion between mass and frame.
- **Piezoelectric instruments:** use piezoelectric crystal to generate charge proportional to acceleration.

By mounting:

- Contact type (vibrometer, accelerometer mounted on machine).
- Non-contact type (laser vibrometer, eddy current probe).

Working principle of vibrometer and accelerometer:

Vibrometer (seismometer) – displacement measurement:

- A mass m is suspended by a spring of stiffness k inside a frame attached to the vibrating body.
- The natural frequency of the instrument is kept very low compared to the vibration frequency ($\omega_n \ll \omega$).
- Under this condition, the relative displacement between mass and frame is approximately equal to the displacement of the vibrating body.
- The relative motion is recorded by a transducer (LVDT, strain gauge, or optical scale).

$$X_{relative} \approx X_{body} \text{ when } \omega \gg \omega_n$$

Accelerometer – acceleration measurement:

- A seismic mass m is mounted on a piezoelectric crystal (or strain gauge) inside a housing.
- The natural frequency of the instrument is kept very high compared to the vibration frequency ($\omega_n \gg \omega$).
- The inertia force on the mass is $F = m\ddot{x}$; this force compresses the crystal, generating a charge proportional to acceleration.
- Output voltage is proportional to acceleration: $V \propto \ddot{x}$.

$$\text{Accelerometer output} \propto \ddot{x} \text{ when } \omega \ll \omega_n$$

Diagram Description:

- **Vibrometer:** A frame containing a heavy mass suspended by a soft spring; relative movement between mass and frame is measured by a scale/transducer.
- **Accelerometer:** A small housing with a seismic mass pressing on a piezoelectric crystal; the crystal generates charge when the mass exerts inertia force during vibration.

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2. ★ Asked 1–2 times – only if time is left:

- **Q.** Measurement of displacement and velocity of vibrating body. **(Priority ★; Asked in: W23, S25; 3 / 4 Marks)**

Answer:

Displacement measurement:

- **Vibrometer / seismometer:** uses low natural frequency suspension; relative displacement between mass and frame equals body displacement.
- **LVDT (Linear Variable Differential Transformer):** core moves with vibrating body; output AC voltage proportional to displacement.
- **Eddy current proximity probe:** non-contact measurement of displacement of conductive target.
- **Capacitive probe:** variation in capacitance due to gap change gives displacement.

Velocity measurement:

- **Velometer:** seismic instrument with natural frequency tuned such that relative velocity is proportional to body velocity.
- **Integration of accelerometer signal:** velocity obtained by integrating acceleration signal.
- **Differentiation of displacement signal:** velocity obtained by differentiating displacement signal.
- **Laser Doppler vibrometer:** measures velocity directly using Doppler shift of laser light.

$v = dx/dt,\ a = dv/dt = d^2x/dt^2$

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3. Also prepare short answers::

- **Q.** Also prepare short answers (new syllabus, not yet asked): condition monitoring, sensors and data acquisition, FFT / frequency-domain analysis, time-domain analysis, vibration signatures for faults.

Answer:

Condition monitoring:

- **Condition monitoring:** Continuous or periodic measurement of machine parameters (vibration, temperature, pressure, oil debris) to detect incipient faults.
- **Purpose:** Predict failures, schedule maintenance, reduce downtime, improve safety.
- **Techniques:** Vibration monitoring, thermography, oil analysis, acoustic emission, motor current signature analysis.
- **Benefits:** Moves from reactive to predictive maintenance; extends machine life.

Sensors and data acquisition:

Sensors used in vibration monitoring:

- **Accelerometer (piezoelectric):** acceleration measurement, wide frequency range.
- **Velometer:** velocity measurement.
- **Proximity probe (eddy current):** displacement measurement, non-contact.
- **Strain gauge:** strain and force measurement.

Data acquisition system (DAQ):

- **Signal conditioning:** amplifies and filters sensor output.
- **Analog-to-digital converter (ADC):** converts analog signal to digital.
- **Sampling:** records discrete data points at a specified sampling rate.
- **Storage and processing:** data stored for analysis (FFT, time-domain statistics).

$f_s \geq 2f_{max}$ (Nyquist criterion)

FFT / frequency-domain analysis:

- **FFT (Fast Fourier Transform):** An algorithm that converts a time-domain signal into its frequency-domain components.
- **Purpose:** Identifies the frequencies present in a vibration signal and their amplitudes.
- **Application:** Fault diagnosis — unbalance shows peak at 1× running speed; misalignment at 2×; bearing defects at high-frequency harmonics.
- **Advantages:** Quickly identifies dominant frequencies, enables spectral comparison, detects faults that are not obvious in time domain.

  $X(f) = \int x(t)\, e^{(-j2\pi ft)}\, dt \quad (\text{limits } -\infty \text{ to } \infty)$

Time-domain analysis:

- **Time-domain analysis:** Examines vibration signal as a function of time.
- **Parameters:** Peak amplitude, RMS value, crest factor, kurtosis, zero-crossing rate.
- **RMS:** Indicates overall vibration severity.

  $x_RMS = \sqrt{[(1/T) \int x^2(t)\, dt]} \quad (\text{limits } 0 \text{ to } T)$
- **Crest factor:** Ratio of peak to RMS; increases with impulsive faults (e.g., bearing defects).
- **Use:** Simple, fast, and effective for overall condition monitoring.

Vibration signatures for faults:

Fault	Vibration Signature
Unbalance	Large peak at 1× running speed in frequency spectrum
Misalignment	Prominent 2× running speed component, axial vibration
Bearing defect	High-frequency impulses, harmonics, BPFO/BPFI frequencies
Looseness	Many harmonics of running speed, sub-harmonics
Gear fault	Sidebands around gear mesh frequency (GMF)
Resonance	Large amplitude at natural frequency

- Overall vibration level increases with fault severity.
- Spectral trend analysis over time helps predict failure.
- Envelope analysis is used for early bearing defect detection.

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4. Repeated questions – do these exact questions first

1. Repeated Questions:

- **Q.** Four masses A, B, C, D (200, 300, 400, 200 kg) balancing in planes X and Y **(W25 and W22)**

Answer:

Given:

Masses and radii:

m_A = 200 kg, r_A = 0.2 m

m_B = 300 kg, r_B = 0.15 m

m_C = 400 kg, r_C = 0.25 m

m_D = 200 kg, r_D = 0.3 m

Angles measured from mass A:

θ_A = 0°, θ_B = 90°, θ_C = 180°, θ_D = 270°

Plane positions from A:

A = 0, B = 0.3 m, C = 0.6 m, D = 0.9 m

Balancing planes: X at 0.15 m, Y at 0.75 m from A.

Balancing radius r_X = r_Y = 0.2 m.

- **To Find:** Balancing masses m_X, m_Y and their angular positions.

Formula:

$$\Sigma mr = 0,\ \Sigma mrl = 0$$

Solution:

Force components mr:

$$\text{A: } 200(0.2) = 40 \text{ at } 0° \rightarrow (40, 0)$$
$$\text{B: } 300(0.15) = 45 \text{ at } 90° \rightarrow (0, 45)$$
$$\text{C: } 400(0.25) = 100 \text{ at } 180° \rightarrow (-100, 0)$$
$$\text{D: } 200(0.3) = 60 \text{ at } 270° \rightarrow (0, -60)$$

Sum of existing forces: (−60, −15).

Let X vector = (X_x, X_y), Y vector = (Y_x, Y_y).

$$X_x + Y_x = 60,\ X_y + Y_y = 15 \quad (1)$$

Couple components mrl:

$$\text{A: } 0$$
$$\text{B: } 45(0.3) = 13.5 \text{ at } 90° \rightarrow (0, 13.5)$$
$$\text{C: } 100(0.6) = 60 \text{ at } 180° \rightarrow (-60, 0)$$
$$\text{D: } 60(0.9) = 54 \text{ at } 270° \rightarrow (0, -54)$$

Sum of existing couples: (−60, −40.5).

$$0.15X_x + 0.75Y_x = 60,\ 0.15X_y + 0.75Y_y = 40.5 \quad (2)$$

Solving (1) and (2):

$$Y_x = 85,\ X_x = -25,\ Y_y = 63.75,\ X_y = -48.75$$

Magnitudes:

$$X = \sqrt{((-25)^2 + (-48.75)^2)} = 54.79 \text{ kg·m}$$
$$Y = \sqrt{(85^2 + 63.75^2)} = 106.25 \text{ kg·m}$$

Balancing masses at r = 0.2 m:

$$m_X = 54.79/0.2 = 273.95 \text{ kg}$$
$$m_Y = 106.25/0.2 = 531.25 \text{ kg}$$

Angular positions:

$$\theta_X = \tan^{-1}(-48.75/-25) = 242.8°$$

$\theta_Y = \tan^{-1}(63.75/85) = 36.9°$

Final Answer:

$m_X \approx 274\ \text{kg at } 242.8°,\ m_Y \approx 531\ \text{kg at } 36.9°$

Diagram Description: Four masses A, B, C, D in parallel planes spaced along the shaft, with balancing masses in planes X and Y. The force polygon and couple polygon are drawn to close with the balancing vectors.

- **Q.** Turning moment diagram with areas +52, –124, +92, –140, +85, –72, +107 **(W25 and S22)**

Answer:

Given:

Areas of turning moment diagram above/below mean torque:

+52, −124, +92, −140, +85, −72, +107 mm²

- **Scale:** 1 mm = 100 N·m (vertical), 1 mm = 1° (horizontal)
- Mean speed, N = 200 rpm
- Coefficient of fluctuation of speed, C_s = 0.02
- Radius of gyration, k = 0.5 m
- **To Find:** Mass of flywheel, m.

Formula:

$\Delta E = \max \text{ excess energy} - \min \text{ excess energy}$

$1\ \text{mm}^2 = 100 \times \pi/180 = 1.745\ \text{J}$

$\Delta E = mk^2\omega^2 C_s,\ \omega = 2\pi N/60$

Solution:

Cumulative energy after each area:

$0 \rightarrow +52 \rightarrow -72 \rightarrow +20 \rightarrow -120 \rightarrow -35 \rightarrow -107 \rightarrow 0$

Maximum excess energy:

$E_max = +52\ \text{mm}^2$

Minimum excess energy:

$E_min = -120\ \text{mm}^2$

Fluctuation of energy:

$\Delta E = (52 - (-120)) = 172\ \text{mm}^2$

$\Delta E = 172 \times 1.745 = 300.1\ \text{J}$

Mean angular speed:

$\omega = 2\pi(200)/60 = 20.944\ \text{rad/s}$

Flywheel mass:

$m = \Delta E / (k^2\omega^2 C_s)$

$m = 300.1 / ((0.5)^2(20.944)^2(0.02))$

$m = 300.1 / (0.25 \times 438.65 \times 0.02)$

$m = 300.1 / 2.193 = 136.8\ \text{kg}$

Final Answer:

$m \approx 136.8\ \text{kg} \approx 137\ \text{kg}$

Diagram Description: A turning moment diagram with mean torque line; positive areas +52, +92, +85, +107 and negative areas −124, −140, −72. Cumulative energy curve rises to a maximum after +52 and falls to a minimum after −140, giving ΔE = 172 mm².

- **Q.** Punching press, 750 holes/hour, 14000 Nm **(W23 and S25)**

Answer:

Punching Press Numerical — Formula Bank

- **Energy required per hole:** E_hole = 14000 N·m
- **Number of holes per hour:** 750
- **Power of motor:**
  $P = E_hole \times 750 / 3600 \ W$
- **Energy supplied by flywheel during punching:**
  $\Delta E = E_hole - E_motor \text{ during punching}$
- **Flywheel mass:**
  $\Delta E = mk^2\omega^2 C_s$
- **Speed fluctuation:**
  $C_s = (N_1 - N_2)/N$

- **Q.** Spring 350 N/m, mass 0.35 kg, damping 14 Ns/m **(W24 and S26)**

Answer:
[Answer not found in the answer chat – to be added]

- **Q.** Mass 30 kg, k = 20 kN/m, damping factor 0.25 **(W24 and S26)**

Answer:

Given:

- Mass, m = 30 kg
- Spring stiffness, k = 20 kN/m = 20,000 N/m
- Damping factor, ζ = 0.25
- **To Find:** c_c, ω_n, ω_d, logarithmic decrement δ, successive amplitude ratio.

Formula:

$\omega_n = \sqrt{(k/m)}, \ c_c = 2\sqrt{(km)} = 2m\omega_n$
$\omega_d = \omega_n \sqrt{(1 - \zeta^2)}, \ \delta = 2\pi\zeta / \sqrt{(1 - \zeta^2)}$

Solution:

Natural frequency:

$\omega_n = \sqrt{(20000/30)} = 25.82 \text{ rad/s}$
$f_n = 25.82/2\pi = 4.11 \text{ Hz}$

Critical damping coefficient:

$c_c = 2\sqrt{(20000 \times 30)} = 2\sqrt{600000} = 2(774.6) = 1549.2 \text{ N·s/m}$

Actual damping coefficient:

$c = \zeta c_c = 0.25(1549.2) = 387.3 \text{ N·s/m}$

Damped frequency:

$\omega_d = 25.82\sqrt{(1 - 0.25^2)} = 25.82\sqrt{0.9375} = 25.82(0.9682) = 25.00 \text{ rad/s}$
$f_d = 25.00/2\pi = 3.98 \text{ Hz}$

Logarithmic decrement:

$\delta = 2\pi(0.25)/\sqrt{(1 - 0.25^2)} = 1.5708/0.9682 = 1.622$

Successive amplitude ratio:

$x_1/x_2 = e^{\delta} = e^{1.622} = 5.06$

Final Answer:

$\omega_n = 25.82 \text{ rad/s}, \ c_c = 1549.2 \text{ N·s/m}$

$\omega_d = 25.00\ \text{rad/s},\ \delta = 1.622,\ x_1/x_2 = 5.06$

- **Q.** Slider-crank 150 mm / 600 mm, 60°, 450 rpm **(S23 and S22)**

Answer:

Given:

- Crank radius, r = 150 mm = 0.15 m
- Connecting rod length, l = 600 mm = 0.60 m
- Crank angle, θ = 60°
- Speed, N = 450 rpm
- **To Find:** Velocity and acceleration of piston; angular velocity ω_c and angular acceleration α_c of connecting rod.

Formula:

$\omega = 2\pi N/60,\ \sin\varphi = r\sin\theta / l$

$v = r\omega\,[\sin\theta + r\sin\theta\cos\theta / (l\cos\varphi)]$

$a = r\omega^2\,[\cos\theta + r\cos 2\theta / (l\cos\varphi)]$

$\omega_c = r\omega\cos\theta / (l\cos\varphi),\ \alpha_c = [-(r\omega^2\sin\theta / l) + \sin\varphi\,\omega_c^2] / \cos\varphi$

Solution:

Angular speed of crank:

$\omega = 2\pi(450)/60 = 47.124\ \text{rad/s}$

Obliquity:

$\sin\varphi = 0.15\sin 60° / 0.60 = 0.2165$

$\varphi = 12.50°,\ \cos\varphi = 0.9763$

Piston velocity:

$v = 0.15(47.124)[0.8660 + 0.15(0.8660)(0.5) / (0.60(0.9763))]$

$v = 7.0686[0.8660 + 0.1109]$

$v = 6.91\ \text{m/s}$

Piston acceleration:

$a = 0.15(47.124)^2[0.5 + 0.25(-0.5)/0.9763]$

$a = 333.10[0.5 - 0.1280]$

$a = 123.9\ \text{m/s}^2$

Angular velocity of connecting rod:

$\omega_c = 0.15(47.124)(0.5) / (0.60(0.9763)) = 6.03\ \text{rad/s}$

Angular acceleration of connecting rod:

$\alpha_c = [-0.15(47.124)^2(0.8660)/0.60 + (0.2165)(6.03)^2] / 0.9763$

$\alpha_c = -484.4\ \text{rad/s}^2$

Final Answer:

$v_{piston} \approx 6.91\ \text{m/s},\ a_{piston} \approx 123.9\ \text{m/s}^2$

$\omega_c \approx 6.03\ \text{rad/s},\ \alpha_c \approx 484\ \text{rad/s}^2$ opposite to ω_c

Diagram Description: A slider-crank mechanism with crank radius r = 150 mm, connecting rod l = 600 mm, crank angle 60°, and piston reciprocating along the line of stroke.

- **Q.** Rotating unbalance 525 N at 6000 rpm, damping factor 0.3 **(S25 and S23)**

Answer:

Given:

- Mass of system, m = 100 kg

- Spring stiffness, $k = 1 \times 10^6$ N/m
- Damping factor, $\zeta = 0.3$
- Excitation force amplitude, $F_o = 525$ N
- Speed, N = 6000 rpm
- **To Find:** Amplitude X, transmissibility T_r, transmitted force F_T.

Formula:

$$\omega = 2\pi N/60,\ \omega_n = \sqrt{(k/m)},\ r = \omega/\omega_n$$
$$X = F_0 / \sqrt{[(k - m\omega^2)^2 + (c\omega)^2]}$$
$$T_r = \sqrt{[(1 + (2\zeta r)^2) / ((1 - r^2)^2 + (2\zeta r)^2)]}$$

Solution:

Excitation frequency:

$$\omega = 2\pi(6000)/60 = 628.32 \text{ rad/s}$$

Natural frequency:

$$\omega_n = \sqrt{(1 \times 10^6/100)} = 100 \text{ rad/s}$$

Frequency ratio:

$$r = 628.32/100 = 6.2832$$

Damping coefficient:

$$c = 2\zeta\sqrt{(km)} = 2(0.3)\sqrt{(10^6 \times 100)} = 6000 \text{ N·s/m}$$

Denominator for amplitude:

$$k - m\omega^2 = 10^6 - 100(628.32)^2 = -3.8478 \times 10^7$$
$$c\omega = 6000(628.32) = 3.7699 \times 10^6$$
$$D = \sqrt{((-3.8478 \times 10^7)^2 + (3.7699 \times 10^6)^2)}$$
$$D \approx 3.866 \times 10^7 \text{ N/m}$$

Amplitude:

$$X = 525 / (3.866 \times 10^7) = 1.358 \times 10^{-5} \text{ m}$$
$$X = 0.01358 \text{ mm}$$

Transmissibility:

$$(2\zeta r)^2 = (2 \times 0.3 \times 6.2832)^2 = 14.212$$
$$(1 - r^2)^2 = (1 - 39.478)^2 = 1480.59$$
$$T_r = \sqrt{(1 + 14.212)} / \sqrt{(1480.59 + 14.212)} = 3.900/38.662 = 0.1009$$

Transmitted force:

$$F_T = T_r F_0 = 0.1009(525) = 52.97 \text{ N}$$

Final Answer:

$$X \approx 1.358 \times 10^{-5} \text{ m} = 0.0136 \text{ mm}$$
$$T_r \approx 0.101,\ F_T \approx 53.0 \text{ N}$$

Diagram Description: A mass-spring-damper system excited by a rotating unbalance force $F_o = 525$ N at 6000 rpm. The transmissibility curve shows r = 6.28 > √2, so the system operates in the isolation region with T_r < 1.

- **Q.** Cantilever shaft 50 mm dia, 300 mm long, 100 kg disc **(W25 and S24)**

Answer:

[Answer not found in the answer chat – to be added]

- **Q.** Aeroplane half circle (50 m or 60 m radius) **(W25 and S26)**

Answer:

[Answer not found in the answer chat – to be added]

- **Q.** Ship turbine rotor (steering, pitching, rolling) **(appears almost every year with new numbers)**

Answer:

Given:

- Mass moment of inertia of rotor, I = 4000 kg·m²
- Speed of rotor, N = 3000 rpm
- Ship speed, v = 36 km/h = 10 m/s
- Radius of turn, R = 200 m
- **To Find:** Gyroscopic couple during steering, C.

Formula:

$$\omega = 2\pi N/60,\ \omega_p = v/R,\ C = I\omega\omega_p$$

Solution:

Spin angular velocity:

$$\omega = 2\pi(3000)/60 = 314.16\ \text{rad/s}$$

Precession angular velocity:

$$\omega_p = 10/200 = 0.05\ \text{rad/s}$$

Gyroscopic couple:

$$C = (4000)(314.16)(0.05)$$

$$C = 62{,}832\ \text{N·m}$$

Convert to kN·m:

$$C = 62.83\ \text{kN·m}$$

Final Answer:

$$C \approx 62.8\ \text{kN·m}$$

Diagram Description: A ship with a turbine rotor spinning about the longitudinal axis. During steering, the rotor axis precesses about a vertical axis with ω_p = v/R; the gyroscopic couple acts in a vertical plane and tends to raise or dip the bow.

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