Enrolment No./Seat No_____

GUJARAT TECHNOLOGICAL UNIVERSITY

BE- SEMESTER–VII (NEW) EXAMINATION – WINTER 2024

Subject Code:3171003 Date:22-11-2024

Subject Name: Digital Signal Processing

Time:10:30 AM TO 01:00 PM Total Marks:70

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

			MARKS
Q.1	**(a)**	Without explicitly solving for $X(z)$, find the region of convergence of the z-transform of following sequence: $x[n] = 2^n u[-n]$	**03**
	(b)	Use the z-transform to perform the convolution of the following two sequences: $$h[n] = \begin{cases} \left(\frac{1}{2}\right)^n & for\ 0 \le n \le 2 \\ 0 & else \end{cases}$$ $$x[n] = \delta[n] + \delta[n-1] + 4\delta[n-2]$$	**04**
	(c)	Find the inverse of each of the following z-transform: $$X(z) = \frac{1}{(1-z^{-1})(1-z^{-2})} \quad ; \quad |z| > 1$$	**07**
Q.2	**(a)**	Compute the convolution $y[n] = x[n] * h[n]$. where, $x[n] = \{1,1,0,1,1\}$ and $h[n] = \{1,-2,-3,4\}$ (↑ at the 0 in $x[n]$, ↑ at the 4 in $h[n]$)	**03**
	(b)	The first nonzero value of a finite-length sequence $x[n]$ occurs at index $n = -6$ and has a value $x[-6] = 3$ and the last nonzero value occurs at index $n = 24$ and has a value $x[24] = -4$. What is the index of the first nonzero value in the convolution $$y[n] = x[n] * x[n]$$ and what is its value? What about the last nonzero value?	**04**
	(c)	For following system, determine whether the system is (1) Memoryless (2) Causal (3) Stable (4) Time Invariant and (5) Linear : $T(x[n]) = e^{x[n]}$	**07**
		OR	
	(c)	For following system, determine whether the system is (1) Memoryless (2) Causal (3) Stable (4) Time Invariant and (5) Linear : $T(x[n]) = (\cos \pi n)\, x[n]$	**07**
Q.3	**(a)**	Figure 1 shows pole- zero plots for two different LTI systems $H_1(z)$ and $H_2(z)$. Based on these plots, state whether or not each system is all-pass system.	**03**
	(b)	For each of the following system functions, state whether or not it is a minimum phase system. Justify your answers.	**04**

(1) $H_1(z) = \frac{(1-2z^{-1})\left(1+\frac{1}{2}z^{-1}\right)}{\left(1-\frac{1}{3}z^{-1}\right)\left(1+\frac{1}{3}z^{-1}\right)}$

(2) $H_2(z) = \frac{1-\frac{1}{3}z^{-1}}{\left(1-\frac{j}{2}z^{-1}\right)\left(1+\frac{j}{2}z^{-1}\right)}$

(c) Consider the causal linear shift-invariant filter with system function: **07**

$$H(z) = \frac{1 + 0.875z^{-1}}{(1 + 0.2z^{-1} + 0.9z^{-2})(1 - 0.7z^{-1})}$$

Draw a signal flowgraph for this system using:

1) Direct form I
2) A cascade of first- and second-order systems realized in transposed direct form II

OR

Q.3 **(a)** Figure 2 shows pole- zero plots for two different LTI systems $H_3(z)$ and $H_4(z)$. Based on these plots, state whether or not each system is all-pass system. **03**

(b) When the input to an LTI system is $x[n] = \left(\frac{1}{3}\right)^n u[n] + 2^n u[-n-1]$, the corresponding output is $y[n] = 5\left(\frac{1}{3}\right)^n u[n] - 5\left(\frac{2}{3}\right)^n u[n]$. **04**

Is the system stable? Is it causal?

(c) Consider the causal linear shift-invariant filter with system function: **07**

$$H(z) = \frac{1 + 0.875z^{-1}}{(1 + 0.2z^{-1} + 0.9z^{-2})(1 - 0.7z^{-1})}$$

Draw a signal flowgraph for this system using:

1) Direct form II
2) A cascade of first- and second-order systems realized in direct form II

Q.4 **(a)** Compare IIR and FIR filters. **03**

(b) Write down pros and cons of digital filters. **04**

(c) Design FIR filter (low pass) using rectangular window with passband gain of unity, cutoff frequency of 200 Hz, sampling frequency of 1kHz. Assume the length of impulse response as 7. **07**

OR

Q.4 **(a)** How can one ensure that the results obtained from circular convolution match those obtained from linear convolution when applied to two given time sequences? **03**

(b) Write down difference between DFT and DTFT. **04**

(c) The transfer function of analog filter is $H_a(s) = \frac{3}{(s+2)(s+3)}$ with T=0.1 sec. Design the digital IIR filter using Bilinear Transformation. **07**

Q.5 **(a)** What is the difference between circular convolution and linear convolution? **03**

(b) Determine the following sequence: $y[n] = x[n] \circledast h[n]$ **04**
Where $x[n] = \{\underset{\uparrow}{1},2,3,1\}$ and $h[n] = \{\underset{\uparrow}{4},3,2,2\}$

(c) Derive DIT FFT flow graph for N=4 and hence find DFT of $x[n] = \{1,2,3,4\}$. **07**

OR

Q.5 **(a)** Write down properties of twiddle factor W_N. **03**

(b) Compute the DFT of the sequence which is expressed as follows: **04**

$$x[n] = \begin{cases} \frac{1}{4}, & 0 \le n \le 2 \\ 0, & otherwise \end{cases}$$

(c) Obtain DFT of the following sequence: **07**

$$x[n] = \left\{\frac{1}{2}, \frac{1}{2}, \frac{1}{2}.\frac{1}{2}, 0,0,0,0\right\}$$

Using the decimation in frequency FFT algorithm.

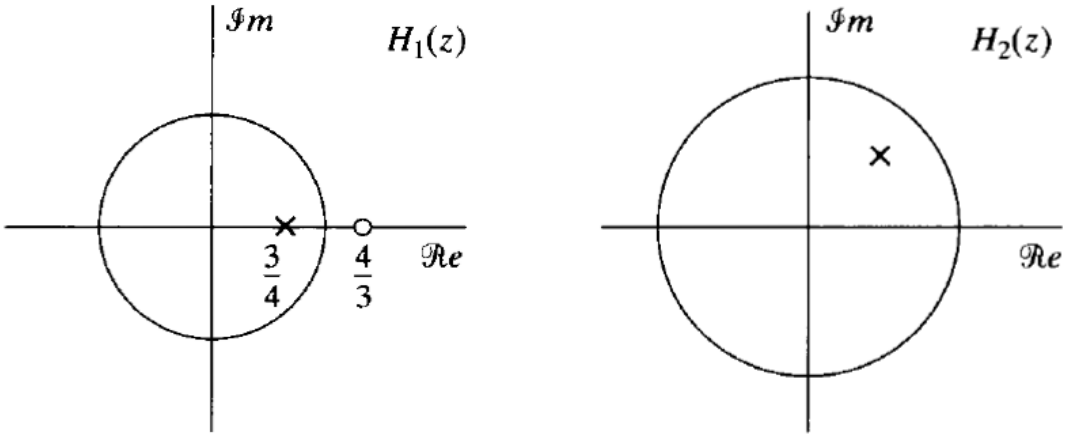

Figure: 1

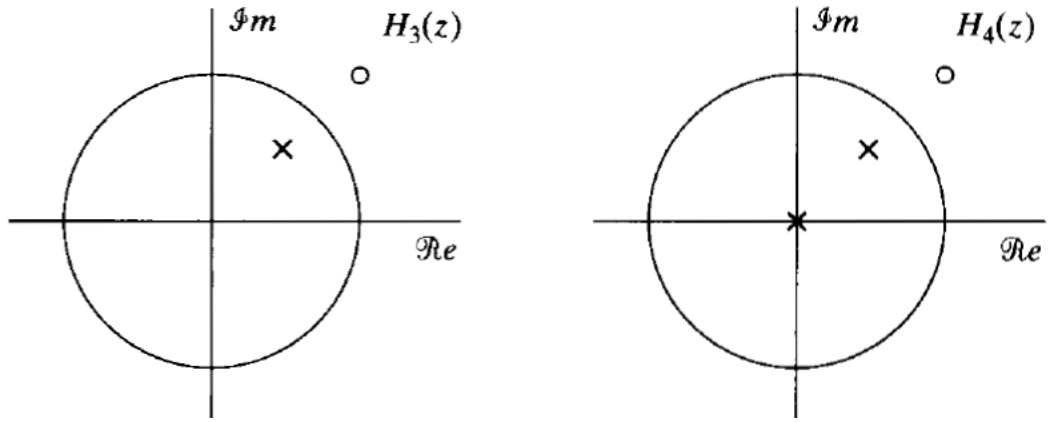

Figure: 2

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